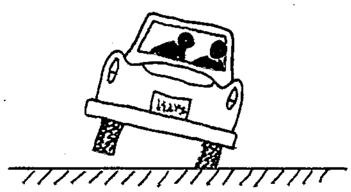
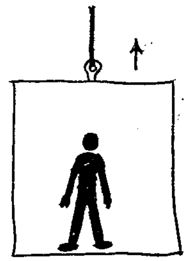
RELATIVITY

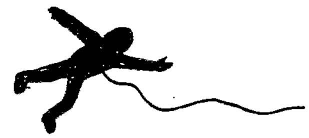
<u>inertial frame of reference</u> - any frame of reference in which Newton's first law of motion holds (isolated object does not accelerate).



inside car rounding a curve



inside accelerating elevator



in deep space away from gravitational fields

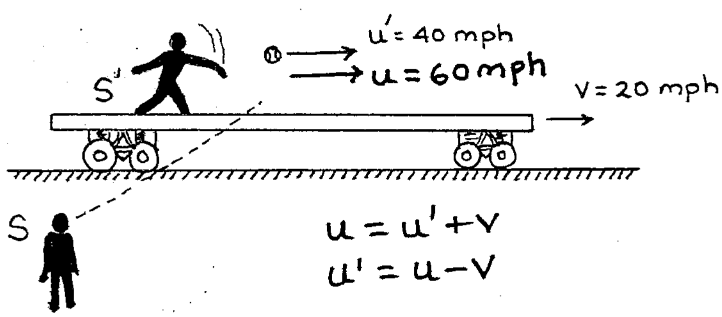
non-inertial reference frames

inertial reference frame

<u>principle of classical relativity</u> - the laws of (Newtonian) mechanics are the same in all inertial reference frames

<u>addition of velocities (classical)</u>

2 inertial frames of reference



$u = u' + V$

$u' = u - V$

S = frame of reference of man standing at station

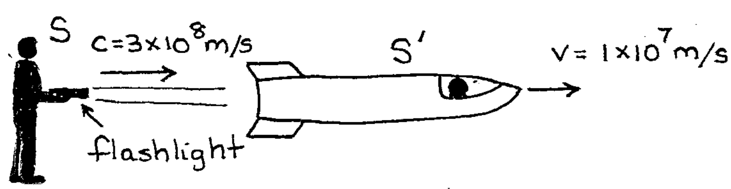
S′ = frame of reference of man on flatcar

u′ = velocity of baseball as measured by thrower in S′ (u = 40 mph)

u = velocity of baseball as measured by man at station in S (u′ = u+V)

$$\boxed{u = u' + V = 40\,\text{mph} + 20\,\text{mph} = 60\,\text{mph}}$$

or



$$\boxed{c' = c - V = 2.9\times10^{8}\,\text{m/s}}$$

* classical relativity and addition of velocities "makes sense", but it is not totally correct

$\boxed{\text{Maxwell's Equations}}$ predict a speed of 3.0×10^8 m/s for light in a vacuum -- but relative to what reference frame?

* any <u>speed</u> must be referenced to some reference frame in order for it to have any meaning

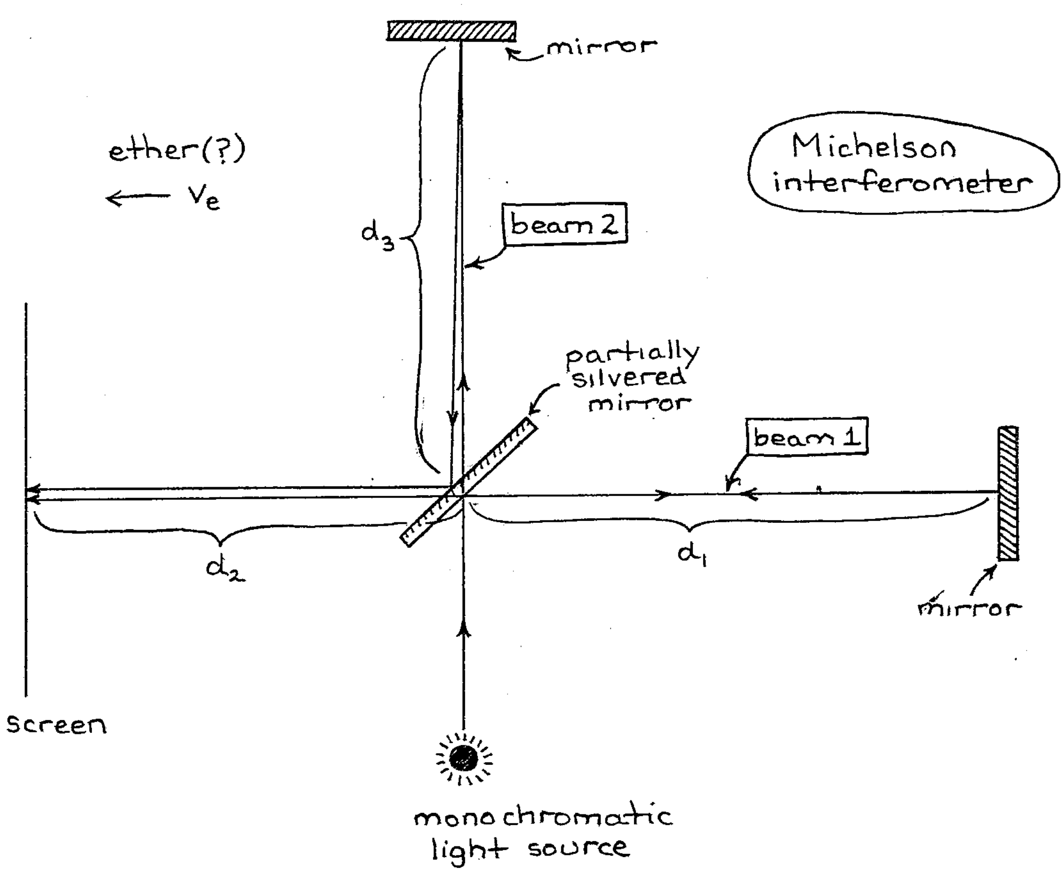
sound travels through air — elastic waves travel through a spring — surface waves travel across the surface of the water

media

* from 1870 - 1905 many physicists believed a medium was necessary to conduct electromagnetic radiation; this was called <u>ether</u> or <u>luminiferous ether</u>

<u>Michelson - Morley Experiment</u> (devised by Albert Michelson, American)

* to detect and measure the speed of the ether



t_1 = time for beam 1 to reach screen

$$t_1 = \frac{d_1}{c - V_e} + \frac{d_1}{c+V_e} + \frac{d_2}{c+V_e}$$

t_2 = time for beam 2 to reach screen

$$t_2 = \frac{d_3}{c} + \frac{d_3}{c} + \frac{d_2}{c+V_e}$$

time of arrival difference $\Delta t = t_1 - t_2$

$$\Delta t = \frac{d_1}{c-V_e} + \frac{d_1}{c+V_e} - \frac{d_3}{c} - \frac{d_3}{c} = \frac{d_1 c}{c^2 - V_e^2} - \frac{2d_3}{c}$$

$\boxed{\text{rotate interferometer } 90^\circ \text{ clockwise}}$

$$t_1' = \frac{d_1}{c} + \frac{d_1}{c} + \frac{d_2}{c}$$

$$t_2' = \frac{d_3}{c-V_e} + \frac{d_3}{c+V_e} + \frac{d_2}{c}$$

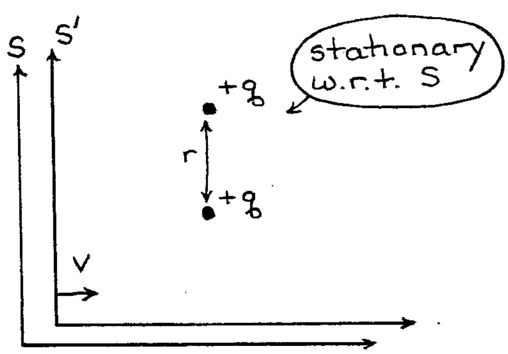
$$\Delta t' = t_1' - t_2' = \frac{2d_1}{c} - \frac{d_3 c}{c^2 - V_e^2}$$

* Since Δt and $\Delta t'$ will be different in general, different interference patterns will be seen for each of the configurations (rotations).

* The shift in fringe patterns observed would allow the computation of V_e

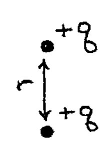
* Result of Michelson-Morley experiment: $\boxed{\text{no fringe shift!!}}$

<u>apparent violation of classical relativity</u>

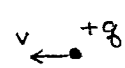


what S sees:

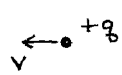
$$F_{rep} = \frac{kq^2}{r^2}$$



what S' sees:



F_{rep} = less than $\frac{kq^2}{r^2}$



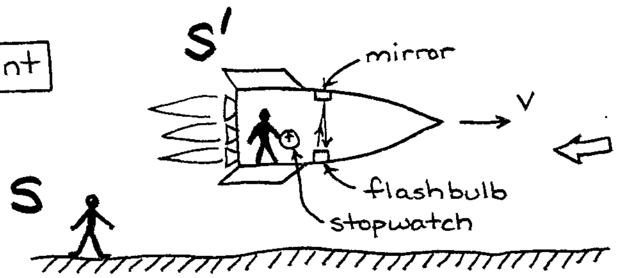
(magnetic force added)

special theory of relativity (A. Einstein, 1905) **STR**.

(1) principle of relativity (postulate): the laws of physics are the same in all inertial frames of reference

(2) constancy of the speed of light (postulate): the speed of light in a vacuum is constant w.r.t. inertial frames of reference

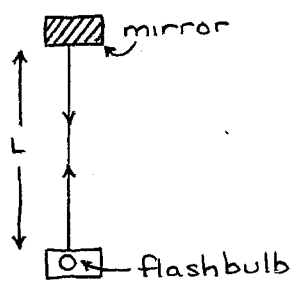
development



two inertial frames of reference

Gedanken Experiment

what the spaceman sees

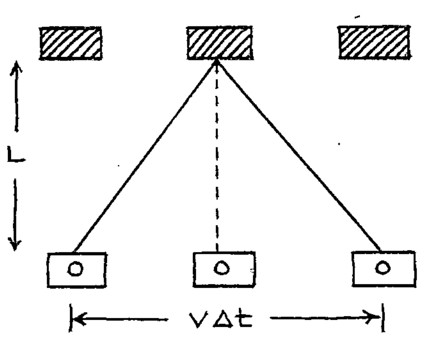


time between flash and light pulse's return to point of origin as measured by spaceman

$$\Delta t_0 = \frac{\text{distance}}{\text{speed}} = \frac{2L}{c}$$

$$\boxed{\Delta t_0 = \frac{2L}{c}}$$

what the earthling sees



$$\Delta t = \frac{\text{distance}}{\text{speed}} =$$

$$\frac{2\sqrt{L^2 + \left(\frac{v\Delta t}{2}\right)^2}}{c} \Rightarrow$$

$$c^2\Delta t^2 = 4\left(L^2 + \frac{v^2\Delta t^2}{4}\right)$$

$$c^2\Delta t^2 - v^2\Delta t^2 = 4L^2$$

but $L = \frac{c\Delta t_0}{2} \rightarrow \quad c^2\Delta t^2 - v^2\Delta t^2 = c^2\Delta t_0^2$

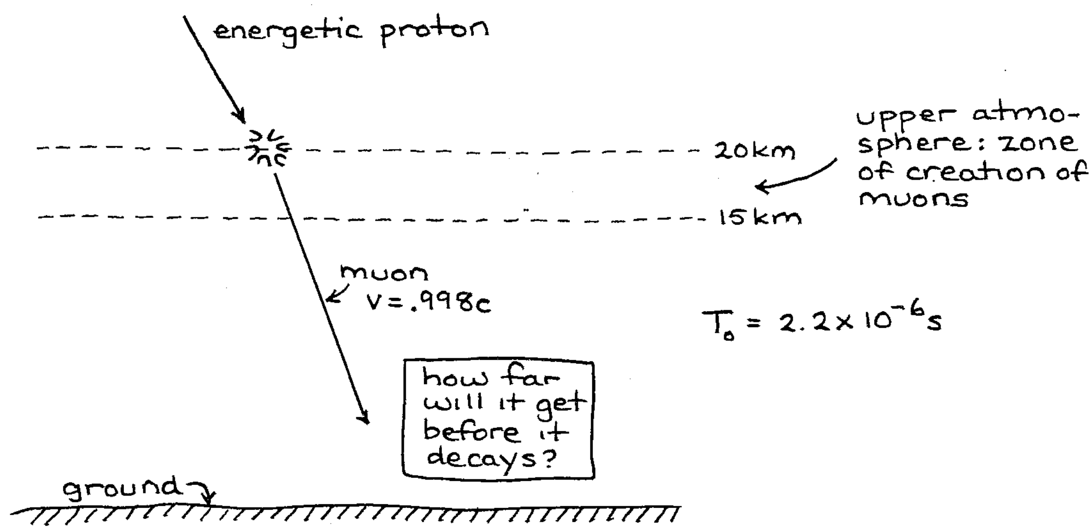
time dilation $\longrightarrow$ $\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}}$

* the factor $\frac{1}{\sqrt{1-v^2/c^2}}$ is called γ

$\gamma = 1$ when $v = 0$
$\gamma = 1.25$ when $v = .6c$
$\gamma = \infty$ when $v = c$

time dilation clocks in moving frames of reference seem to run slow by a factor of γ as observed from a relatively stationary frame

eg mu mesons (muons, μ) created when extremely energetic protons collide with nuclei; they very quickly decay into electrons (or positrons) and neutrinos; average lifetime of a meon in the laboratory is 2.2×10^{-6} s.



* if no time dilation

$$\text{distance} = vT_0 = .998(3\times10^{8}\,\text{m/s})(2.2\times10^{-6}\,\text{s}) = \boxed{.66\,\text{km}}$$

it would never reach the ground

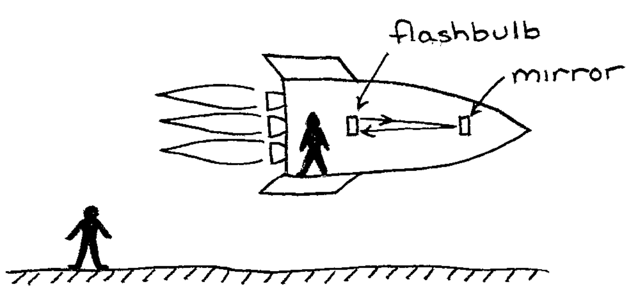
* with time dilation

$$\text{distance} = vT = v(T_0\gamma) = (vT_0)\gamma = (.66\,\text{km})\gamma = \boxed{10.4\,\text{km}}$$

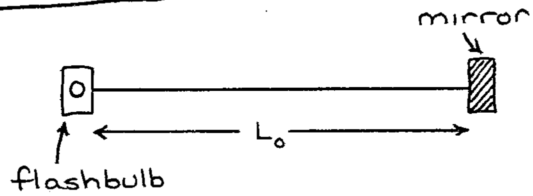
$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-.998^2}} = 15.8$$

much closer!! to ground

development



what the spaceman sees

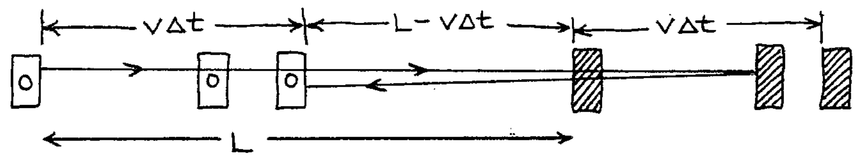


time between flash and return of pulse to point of origin as seen by spaceman

$$\Delta t_o = \frac{\text{distance}}{\text{speed}} = \frac{2L_o}{c}$$

$$\boxed{\Delta t_o = \frac{2L_o}{c}}$$

what the earthling sees



$$\Delta t = \frac{L}{c-v} + \frac{L}{c+v}$$

$$= \frac{2cL}{c^2-v^2} = \frac{2L\gamma^2}{c}$$

$$\boxed{\Delta t = \frac{2L\gamma^2}{c}}$$

* using time dilation ($\Delta t = \Delta t_o \gamma$)

$$\Delta t = \frac{2L\gamma^2}{c} = \Delta t_o \gamma = \frac{2L_o}{c}\gamma \Rightarrow \boxed{L = \frac{L_o}{\gamma}}$$

Lorentz-FitzGerald contraction

eg: an observer sees a spaceship that is 100 m long when at rest; if this ship were passing the observer at 0.5c, how long will the observer measure it to be?

relativistic correction to momentum

$p = m_0 v$ — classical

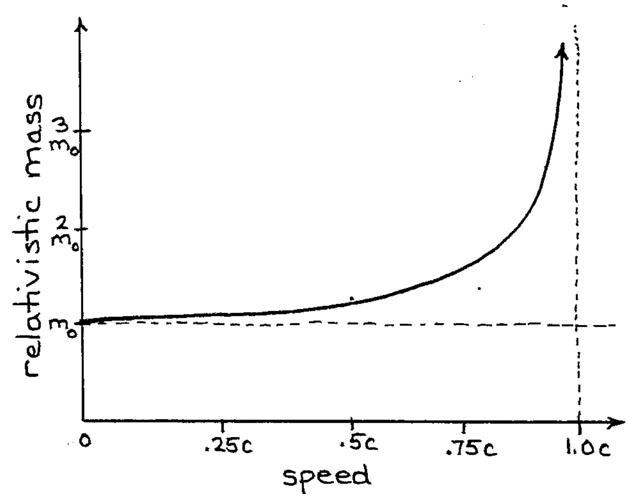
$p = m_0 \gamma v$ — relativistic

conservation of momentum - in any isolated system the total relativistic momentum remains constant

$$p_{rel} = \overbrace{m_0 \gamma}^{\text{relativistic mass} \leftarrow \text{often denoted } m} v$$

m_0: rest mass

$$\text{relativistic mass} = m_0\gamma = \frac{m_0}{\sqrt{1 - v^2/c^2}} = m$$



energy and momentum

* the total energy (relativistic) of a mass can in one sense be thought of as composed of 2 components

m_0 v

$$E = M_0 + K$$

E: total energy (relativistic)

M_0: rest mass energy, $M_0 = m_0 c^2$

K: kinetic energy (relativistic)

$$\boxed{\begin{aligned} E &= mc^2 = m_0\gamma c^2 \\ M_0 &= m_0 c^2 \\ K &= E - M_0 = mc^2 - m_0c^2 \\ &= m_0(\gamma - 1)c^2 \end{aligned}}$$

eg Compute the rest mass energy of a proton

$$M_p = m_p c^2 = (1.67\times10^{-27}\,kg)(3\times10^8\,m/s)^2 = \boxed{1.5\times10^{-10}\,J}$$
$$= \boxed{938\ MeV}$$

* In high energy physics, MeV (million electron volts) is a commonly used energy and mass-energy unit

$M_{electron} = .511$ MeV

$M_{proton} = 938.3$ MeV

$M_{neutron} = 939.6$ MeV

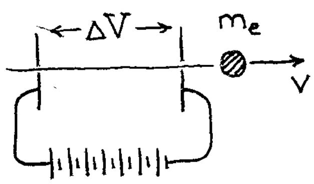
eg How much energy is required to accelerate an electron from rest to .95c?

$$K = M_e(\gamma-1)c^2 = M_e(\gamma-1) = (.511\ MeV)\left(\frac{1}{\sqrt{1-.95^2}}-1\right) = \boxed{1.13\ MeV}$$

What would the classical kinetic energy be?

$$K_{classical} = \tfrac{1}{2}m_e v^2 = \tfrac{1}{2}(m_e c^2)\left(\tfrac{v}{c}\right)^2 = \tfrac{1}{2}(.511\ MeV)(.95)^2 = .23\ MeV$$

eg An electron is accelerated across a 2,000,000 V potential difference. What is its final speed?



$$K = q\Delta V = M_e(\gamma-1)$$

$$\gamma - 1 = \frac{q\Delta V}{M_e}$$

$$\gamma = \frac{qV}{M_e} + 1 = \frac{2\times10^6\,eV}{.511\times10^6\,eV} + 1 = 4.9139$$

$$\frac{1}{\sqrt{1-v^2/c^2}} = 4.9139$$

$$1 - v^2/c^2 = \frac{1}{4.9139^2}$$

$$v^2 = \left(1 - \frac{1}{4.9139^2}\right)c^2 = 8.627\times10^{16}$$

$$\boxed{v = 2.94\times10^8\,m/s = .979\,c}$$

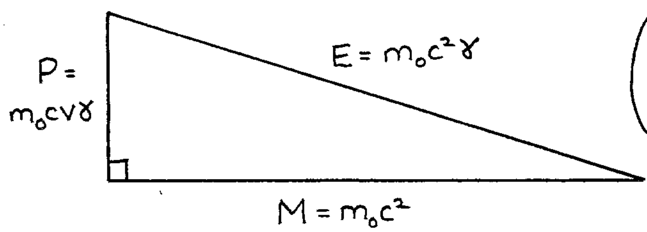
<u>note</u>: relativistic momentum is $p = m_o v \gamma$

define (relativistic) momentum-energy as $P = pc = m_o v c \gamma$

then $P^2 = m_o^2 c^2 v^2 \gamma^2 = M^2 \frac{v^2}{c^2} \gamma^2$

and $M^2 + P^2 = M^2 + M^2 \frac{v^2}{c^2}\gamma^2 = M^2 \underbrace{\left(1 + \frac{v^2}{c^2}\gamma^2\right)}_{\gamma^2} = M^2\gamma^2 = (M\gamma)^2 = E^2$ ← total energy

so $\boxed{E^2 = M^2 + P^2}$ or $E^2 = (m_o c^2)^2 + (m_o v \gamma c)^2$



mass energy and momentum energy "add at right angles" to give total energy

<u>general theory of relativity</u> (A. Einstein, 1915)

* special relativity extended for use in acceleration (esp. gravitational) fields

<u>principle of equivalence</u> - no experiment performed in a closed, accelerating system can distinguish between the effects of a uniform gravitational field and the effects of a uniform acceleration

implications of GTR