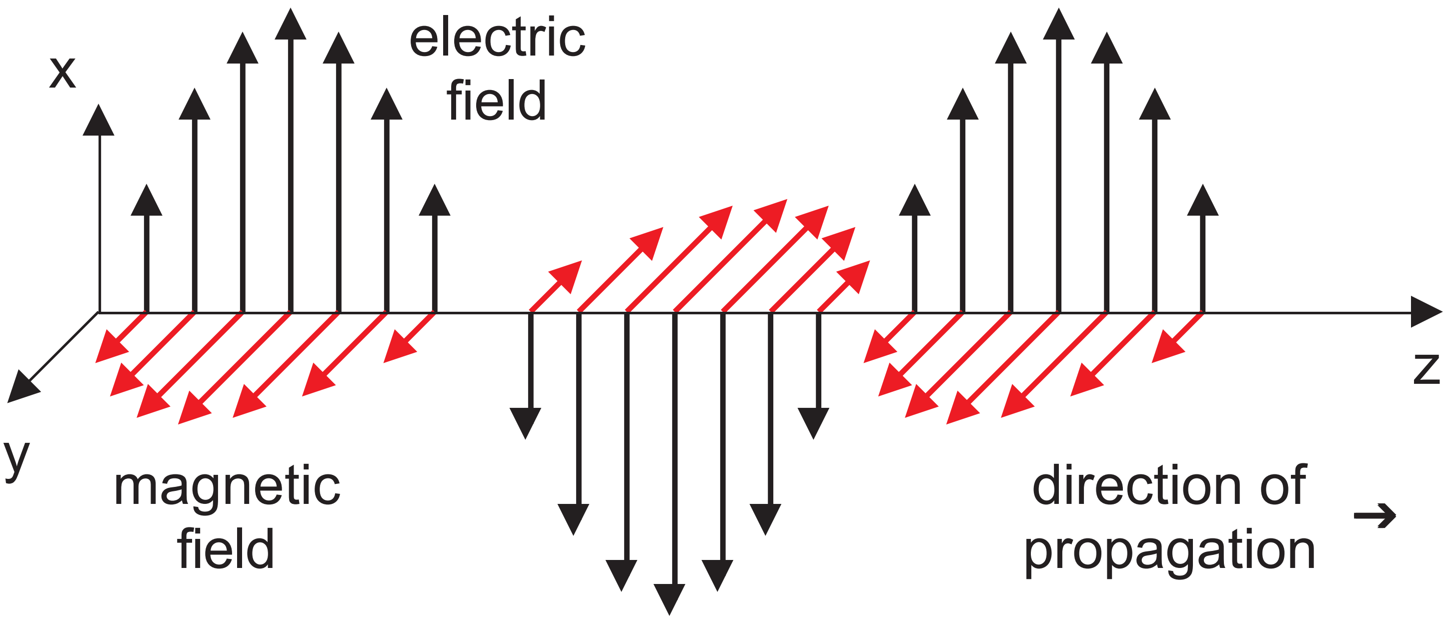
ELECTROMAGNETIC PLANE WAVE

$$\mathbf{E}(z,t) = E_o \sin(kz - \omega t)\mathbf{i}$$
$$\mathbf{B}(z,t) = B_o \sin(kz - \omega t)\mathbf{j}$$

where: ω/k must be $c = (\mu_o \varepsilon_o)^{-1/2}$
and E_o must be $B_o c$

note: $c = 3.0 \times 10^8$ m/s = speed of light in vacuuo



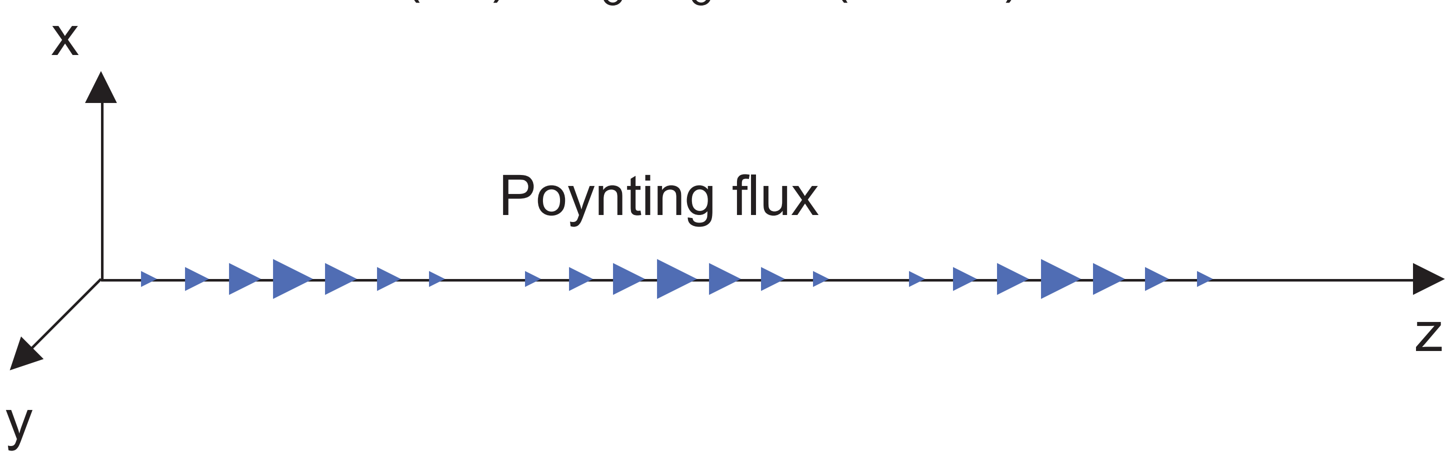
note: The above represents the electromagnetic plane wave specified by the vector fields above as seen at t = 0.

POYNTING FLUX (PLANE WAVE)

The **Poynting flux S** = $(1/\mu_o)$**E x B** for an electromagnetic field is a vector giving the direction of propagation and the time rate at which electromagnetic energy passes through unit area in the direction perpendicular to propagation. In the SI it has units of W/m^2.

For a plane wave in which **E**$(z, t) = E_o \sin(kz - \omega t)$**i** and **B**$(z, t) = B_o \sin(kz - \omega t)$**j**

$$\mathbf{S}(z, t) = \varepsilon_o c E_o^2 \sin^2(kz - \omega t)\mathbf{k}$$



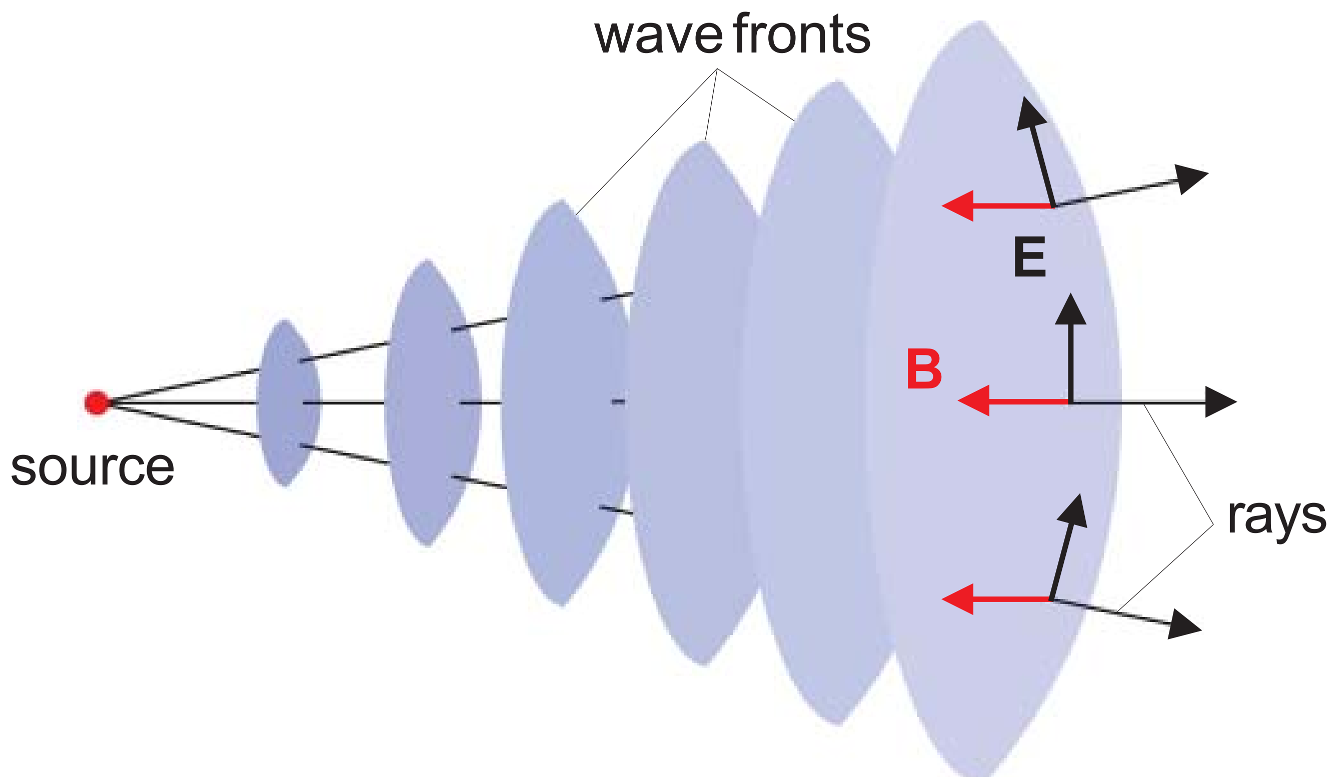
note: The above represents the electromagnetic plane wave specified by the vector fields above as seen at t = 0.

The time average value of this vector anywhere is:

$$\mathbf{S}_{ave} = (1/2)\varepsilon_o c E_o^2 \mathbf{k}$$

ELECTROMAGNETIC RADIATION

Plane wave electromagnetic radiation is one of the simplest solutions to Maxwell's Equations. There are many more solutions (mathematically more complex) that are of importance in optics. A **wave front** is a surface in an electromagnetic wave composed of all points for which the phase is the same. The example below represents an electromagnetic wave for which the wave fronts are not planar.



A **ray** is a line of alignment of Poynting vectors in an electromagnetic wave.

A **beam** is a parallel (or sub-parallel) bundle of rays in an electromagnetic wave.

INDEX OF REFRACTION

The **index of refraction** (n) of a material is the ratio of the speed of light in vacuuo (c) to the speed of light in the material (v).

$$n = c/v$$

Indices of refraction for any materials other than a vacuum will be greater than unity.

material	index of refraction n
vacuum	1.00
air	1.00029
carbon dioxide	1.00045
ice	1.31
water	1.33
ethanol	1.36
fluorite	1.43
fused quartz	1.46
glycerin	1.47
polystyrene	1.49
benzene	1.50
crown glass	1.52
sodium chloride	1.54
flint glass	1.66
aluminum oxide	1.67
zircon	1.92
diamond	2.42

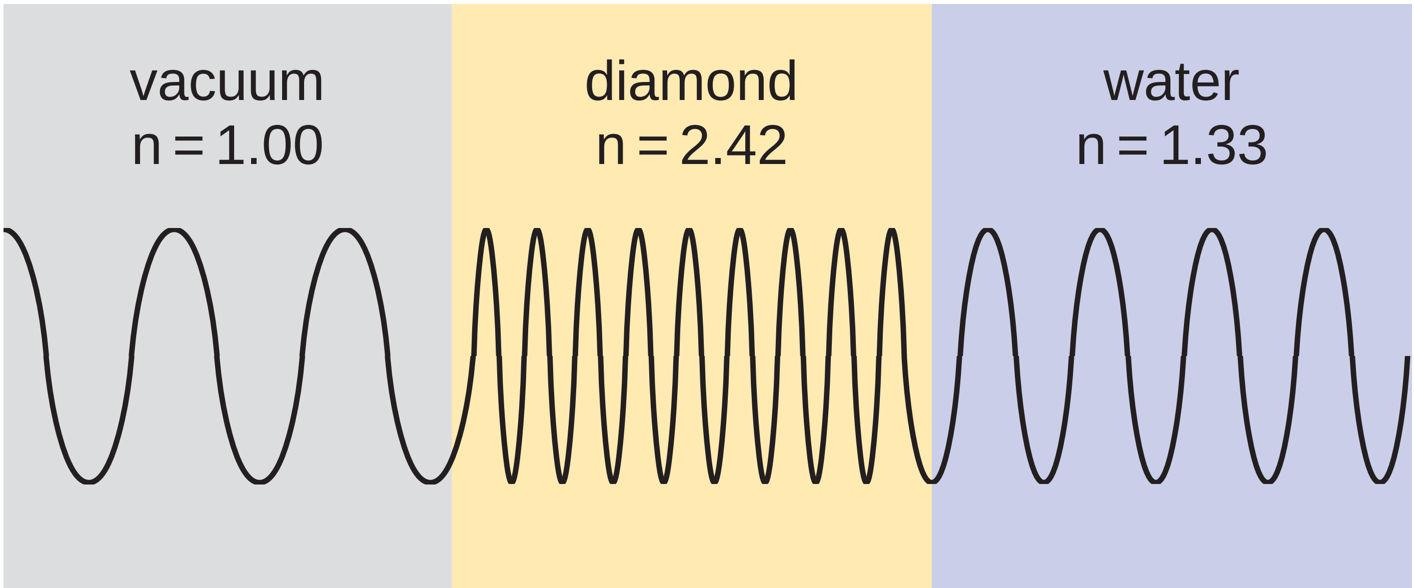
HUYGENS' PRINCIPLE

All points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets. These wavelets then propagate outward through a medium with a speed that is characteristic of that medium. After some time interval has passed, the new position of the wave front is the surface that is tangent to the wavelets.

Christiaan Huygens
(1629-1695)

SPEED AND WAVELENGTH

medium 1	medium 2	medium 3



$v = 3.00 \times 10^8$ m/s	$v = 1.24 \times 10^8$ m/s	$v = 2.25 \times 10^8$ m/s
$f = 5 \times 10^{14}$ Hz	$f = 5 \times 10^{14}$ Hz	$f = 5 \times 10^{14}$ Hz
$\lambda = 600$ nm	$\lambda = 248$ nm	$\lambda = 450$ nm

The wave relation **$v = f\lambda$** is valid in all three media above. The one quantity that cannot change upon passage of the wave from one medium into another is the frequency. Thus $n\lambda$ is the same in all media.

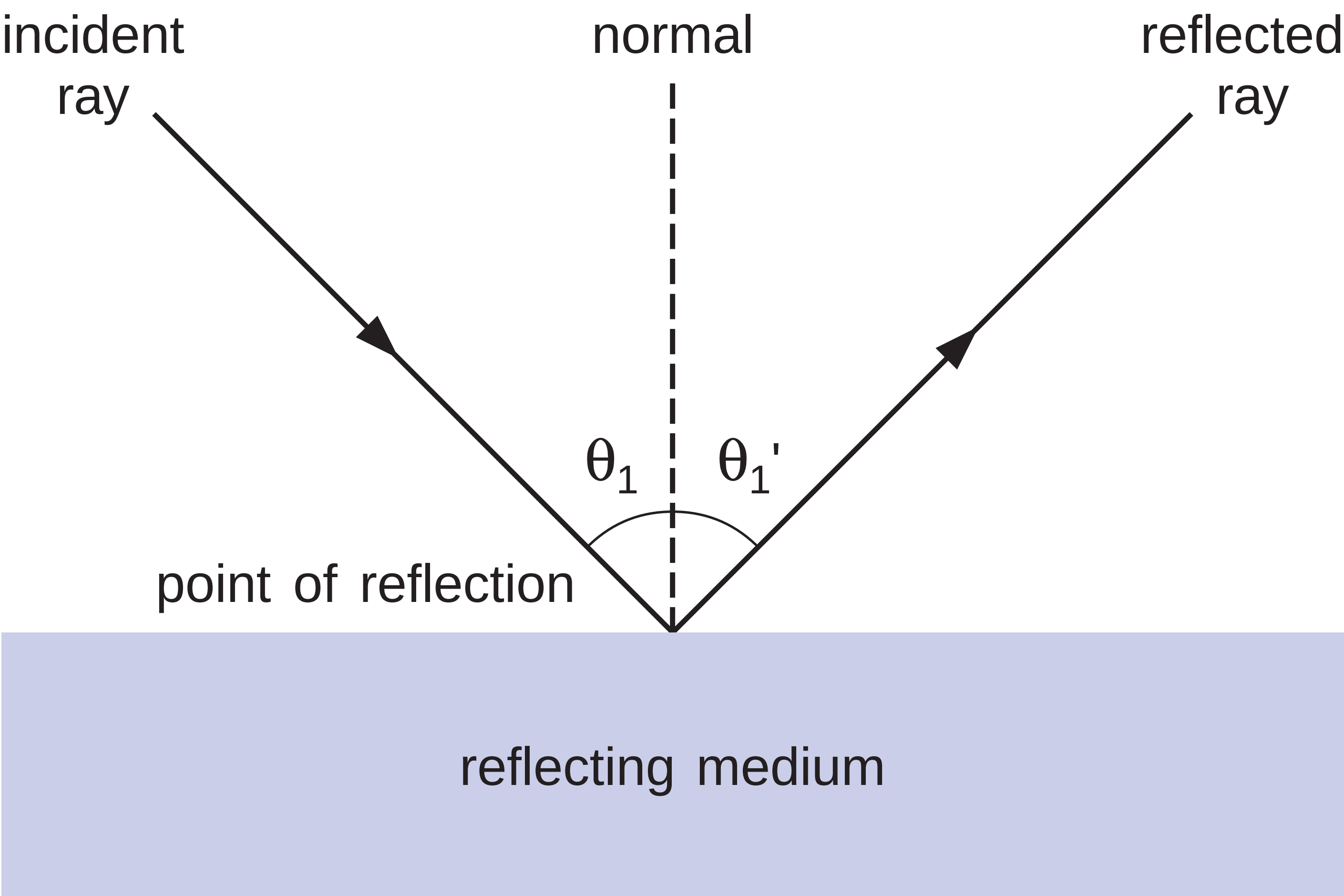
FERMAT'S PRINCIPLE

When a light ray travels between any two points [P and Q], its path is the one that makes the travel time a minimum with respect to small variations in path [local minimum]

Pierre de Fermat
(1601-1665)

REFLECTION

Reflection is the absorption and subsequent emission of light by means of complex electronic vibrations in the atoms of the reflecting medium.

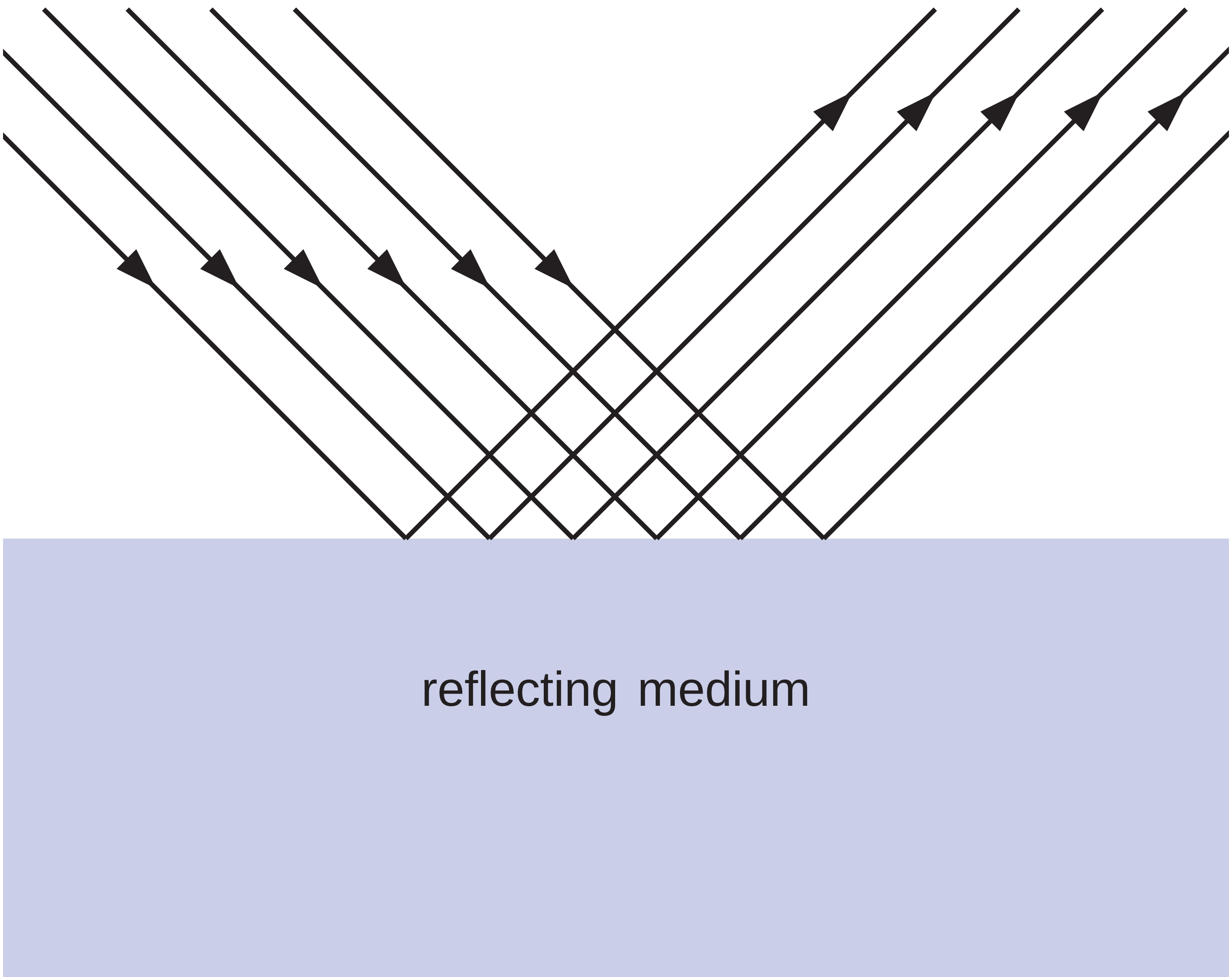


LAW OF REFLECTION

$$\theta_1 = \theta_1'$$

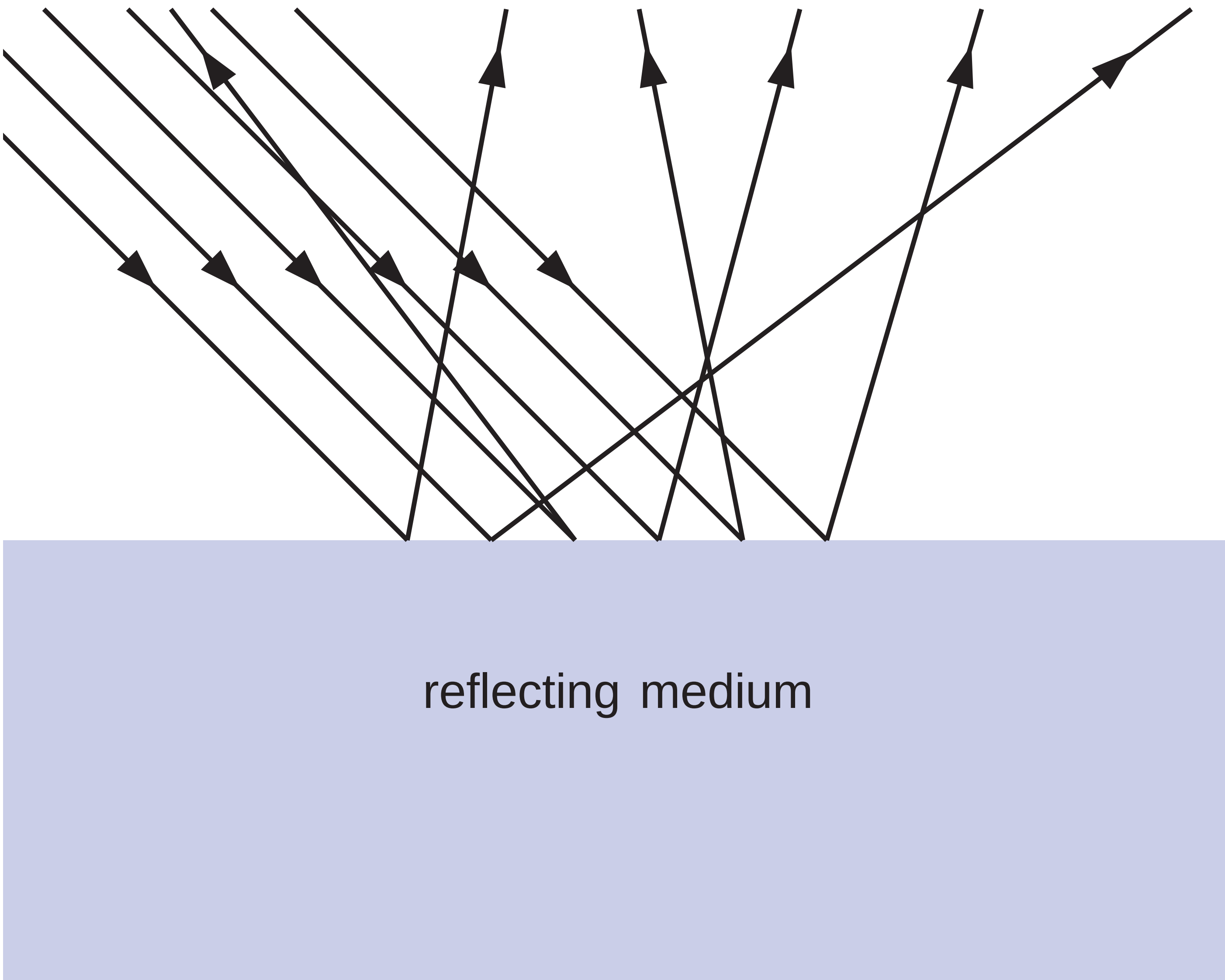
SPECULAR REFLECTION

Reflection from an extremely planar surface in which the reflected rays are parallel. Also called **regular reflection**.



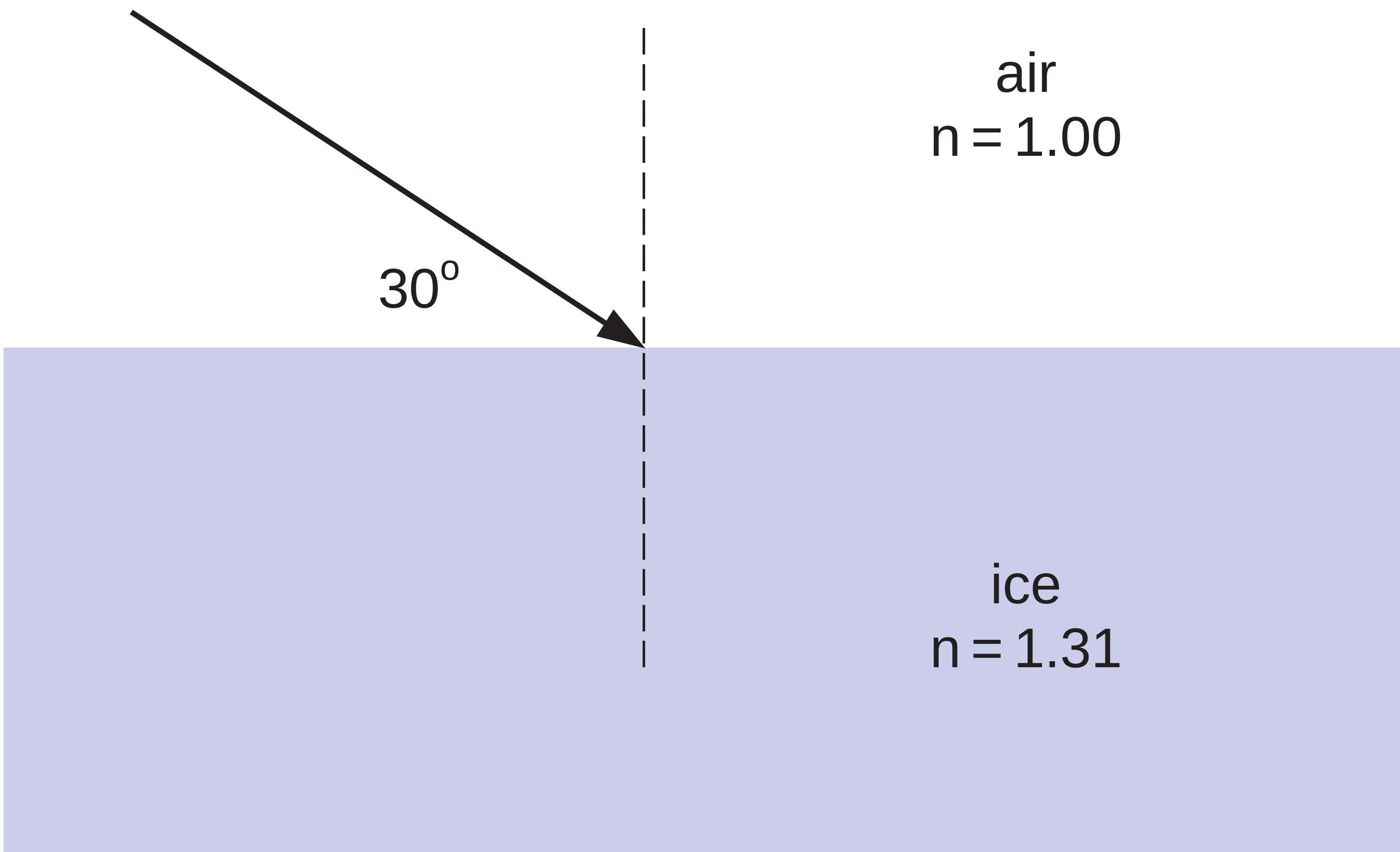
DIFFUSE REFLECTION

Reflection from an irregular surface in which the reflected rays are directionally randomized. Also called **irregular reflection**.



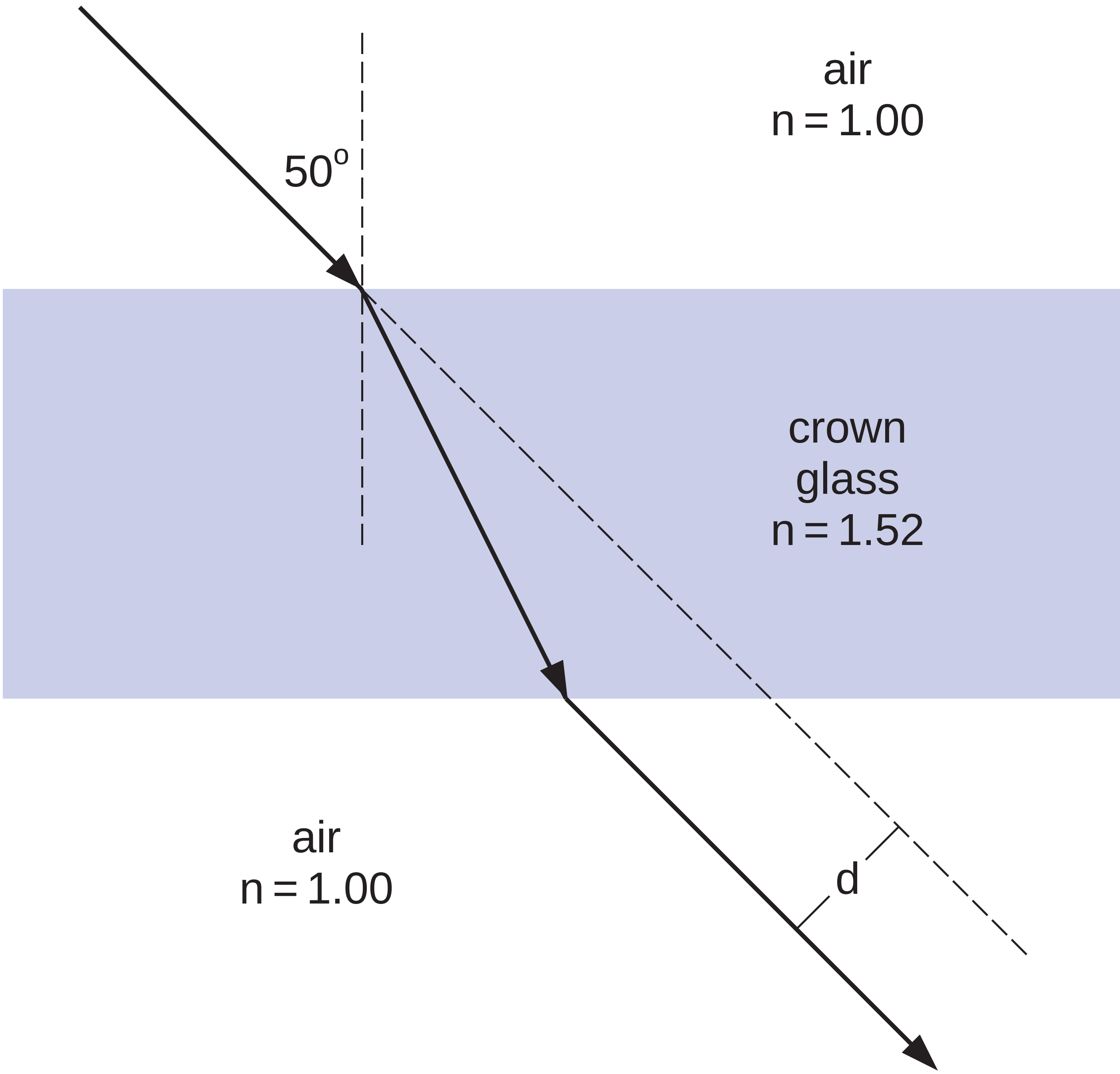
REFRACTION PROBLEM

A beam of light traveling through air strikes the surface of a slab of ice at an angle of 30^o measured from the horizontal. What will be the angle made by the refracted ray as measured from the horizontal?



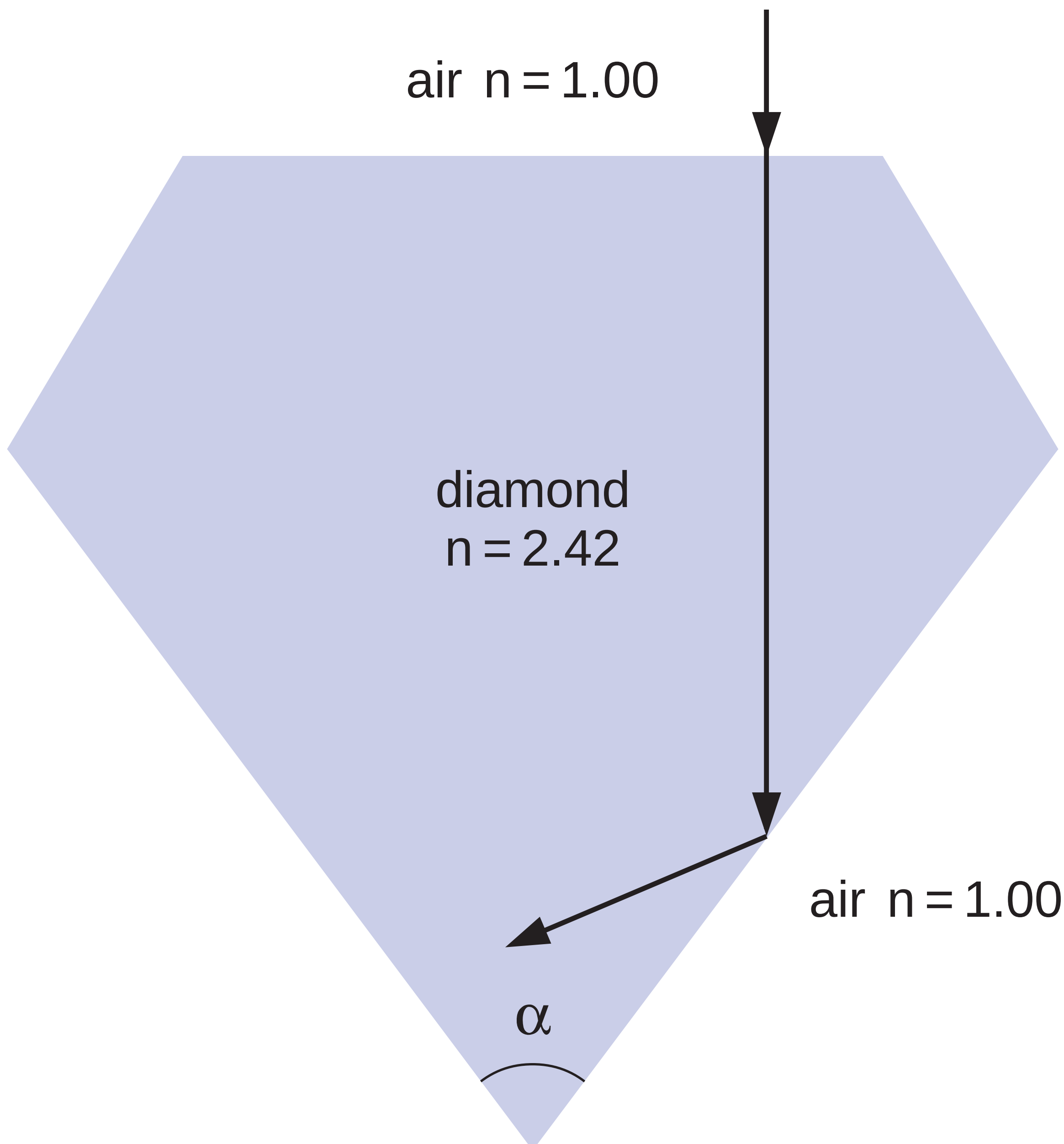
REFRACTION PROBLEM

Assume that the slab of crown glass shown below has a thickness of 4.0 cm. What would be the lateral displacement d of the beam shown passing through it?



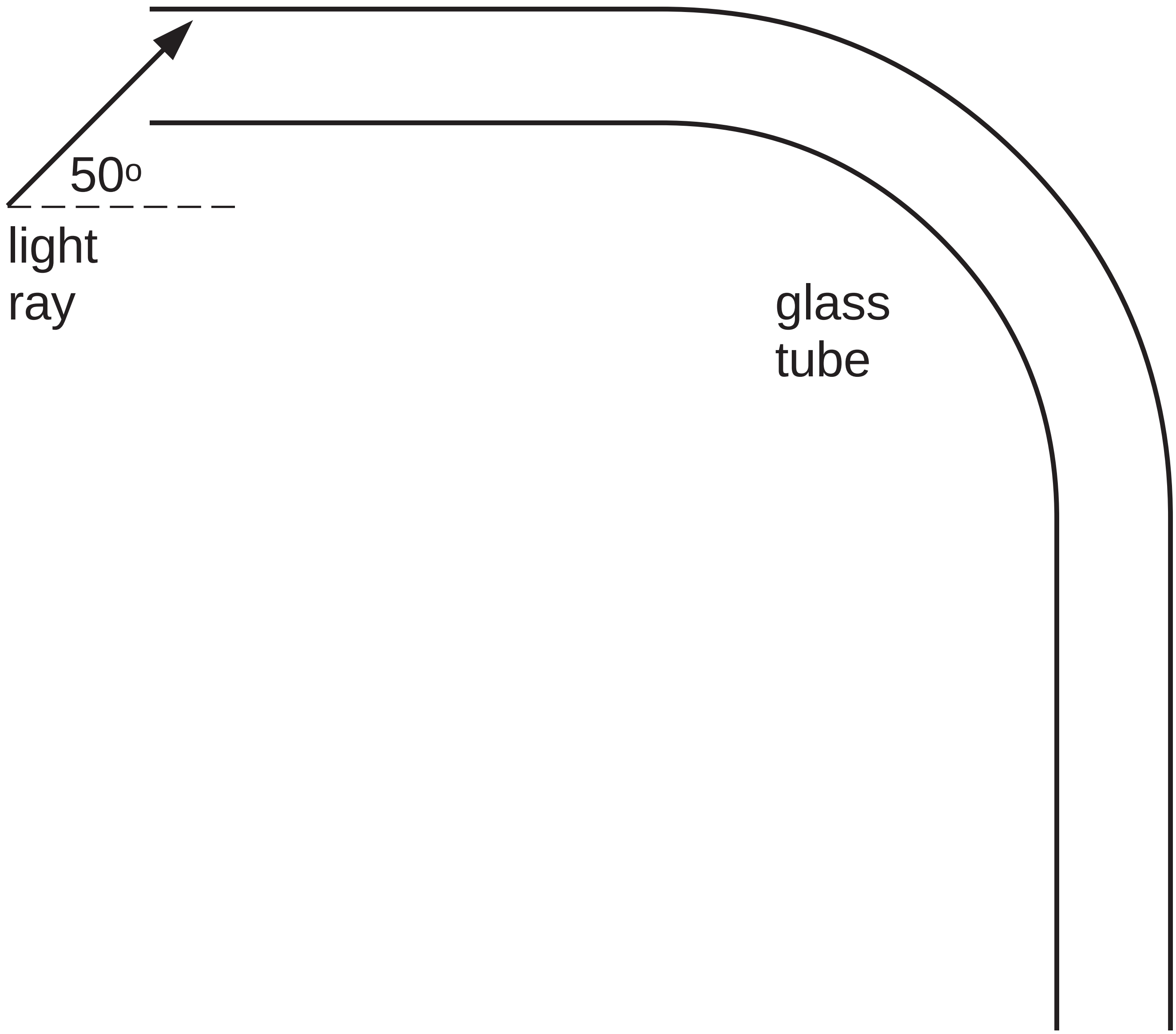
REFRACTION PROBLEM

A diamond cutter plans on shaping the lower portion of the diamond below so that the ray shown is totally internally reflected at the diamond-air interface. What is the largest value of α for which this can happen?



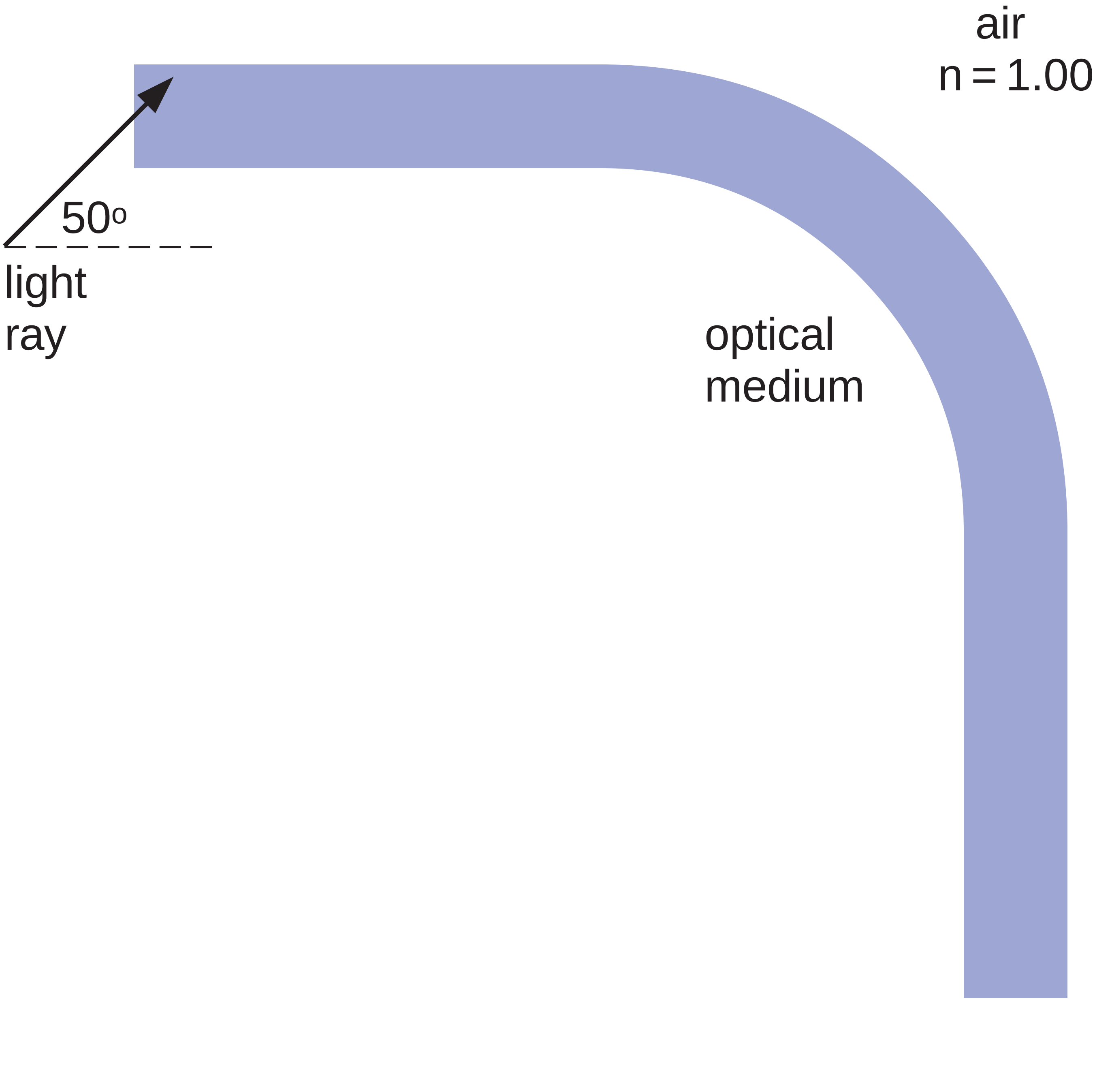
LIGHT CONDUIT

The light conduit shown below is a cylindrical glass tube with circular cross-section. The inner surface of it is coated with a thin layer of highly reflective aluminum whose reflection coefficient is .95.



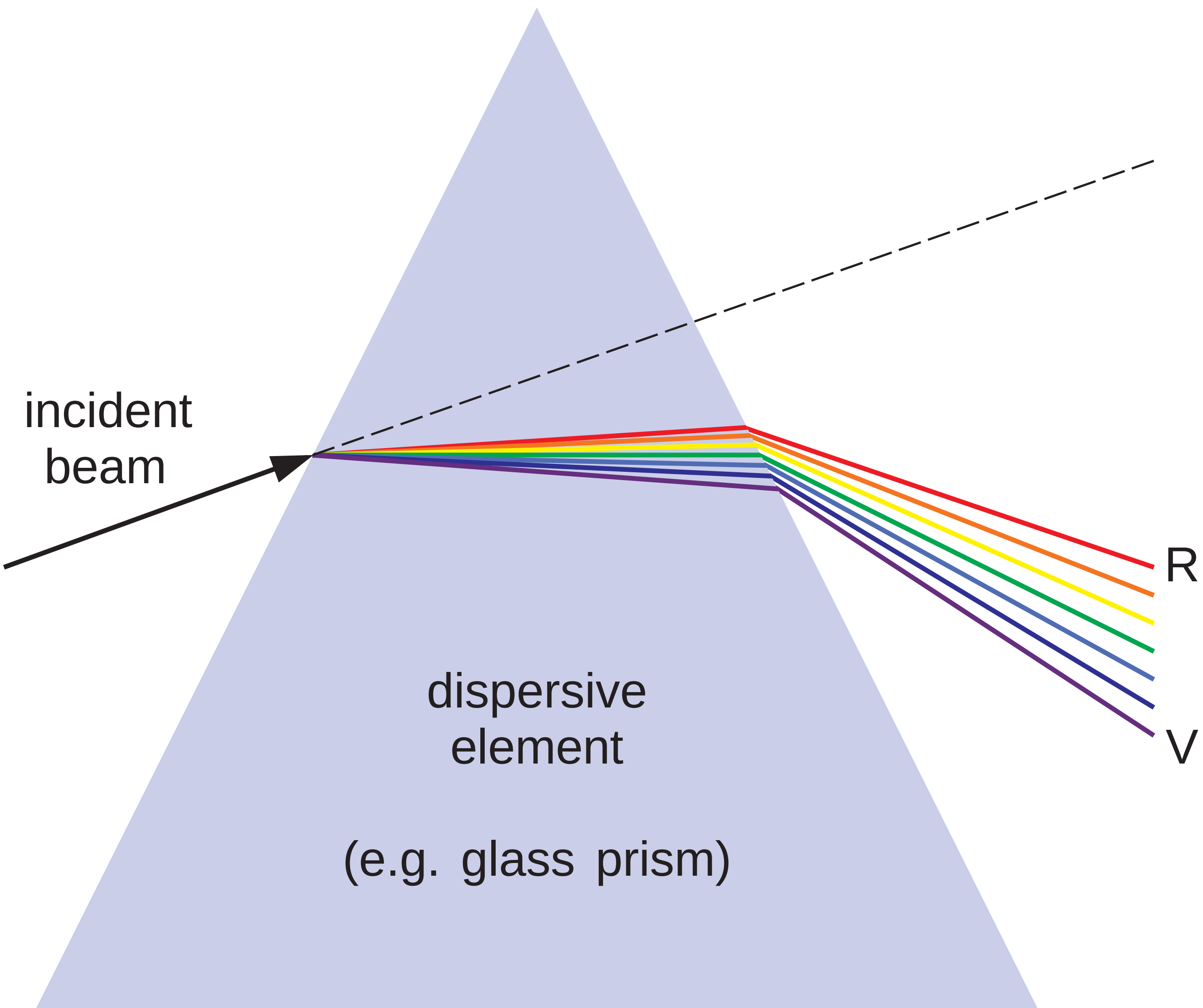
LIGHT CONDUIT

The **fiber-optic** light conduit shown below is an optically clear medium with circular cross-section and index of refraction $n > 1.00$. The reflection coefficient for each T.I.R. reflection is ~ 1.00.



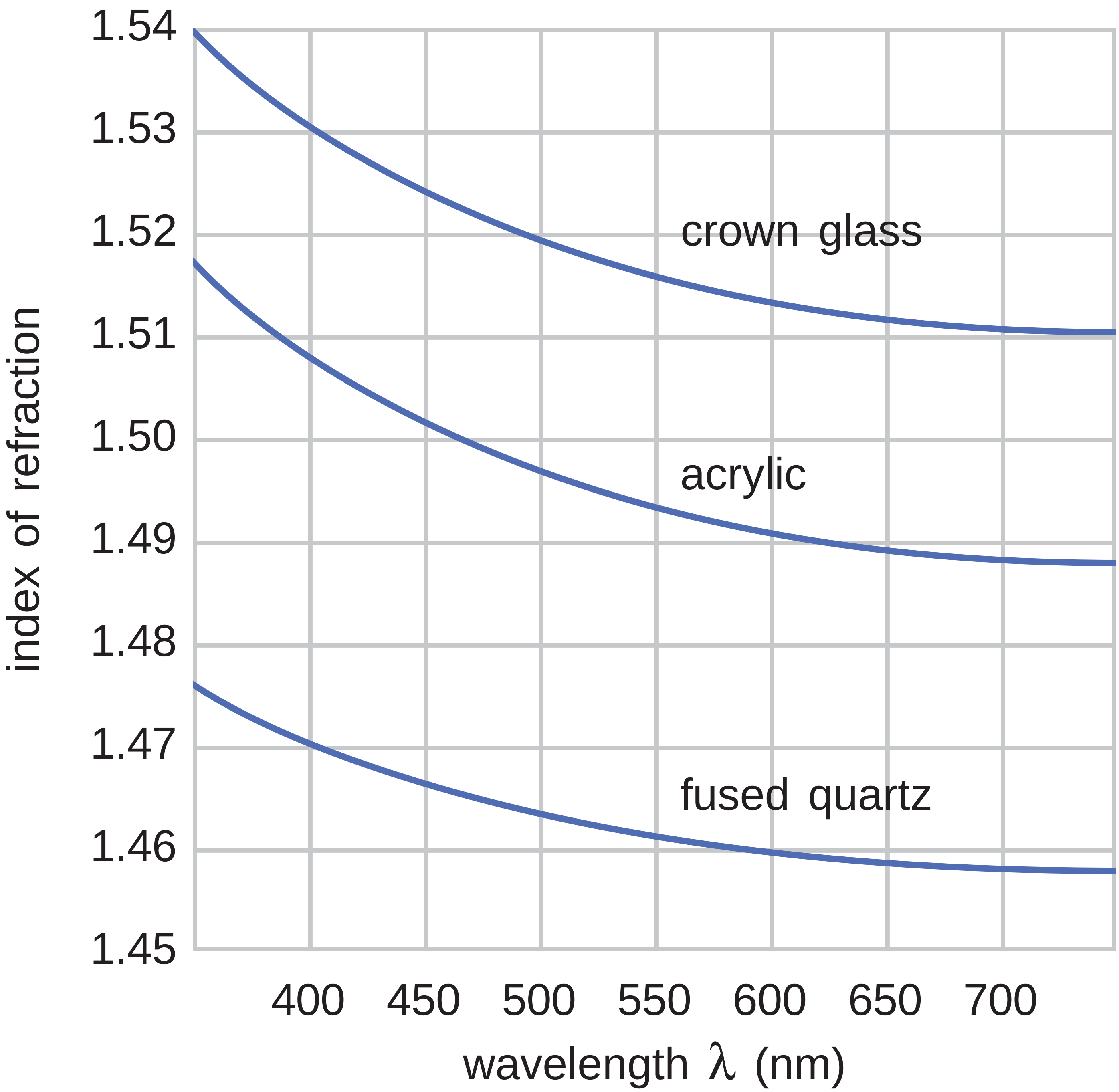
DISPERSION

Dispersion is the phenomenon in which a medium which has a wavelength dependent index of refraction $n(\lambda)$ causes light of different wavelengths passing through it to be: (1) refracted in different directions, or (2) travel with different speeds. **Dispersion** is differential refraction.



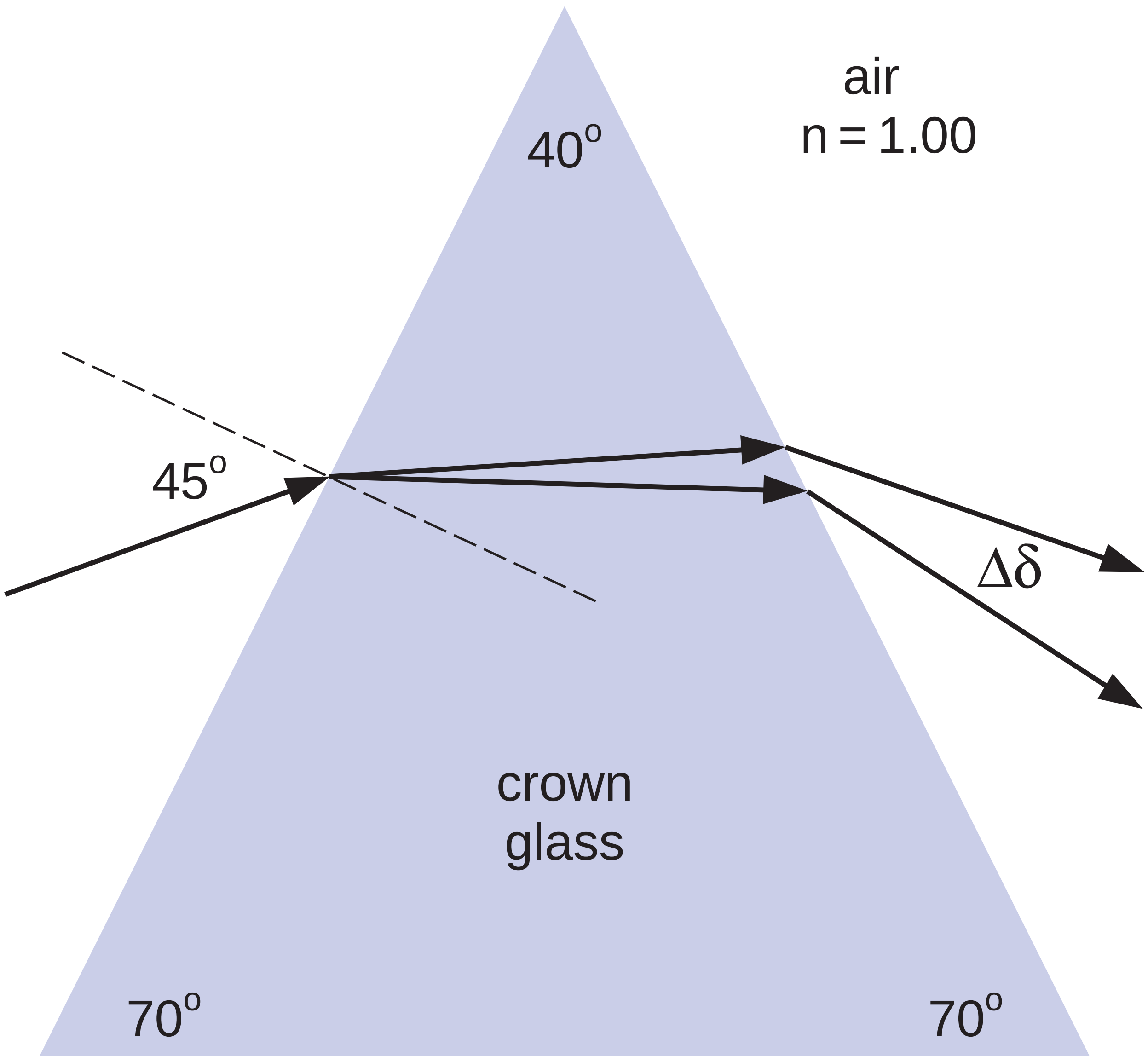
DISPERSIVE MEDIA

Below are graphs of index of refraction versus wavelength for the dispersive media: crown glass, acrylic, and fused quartz.



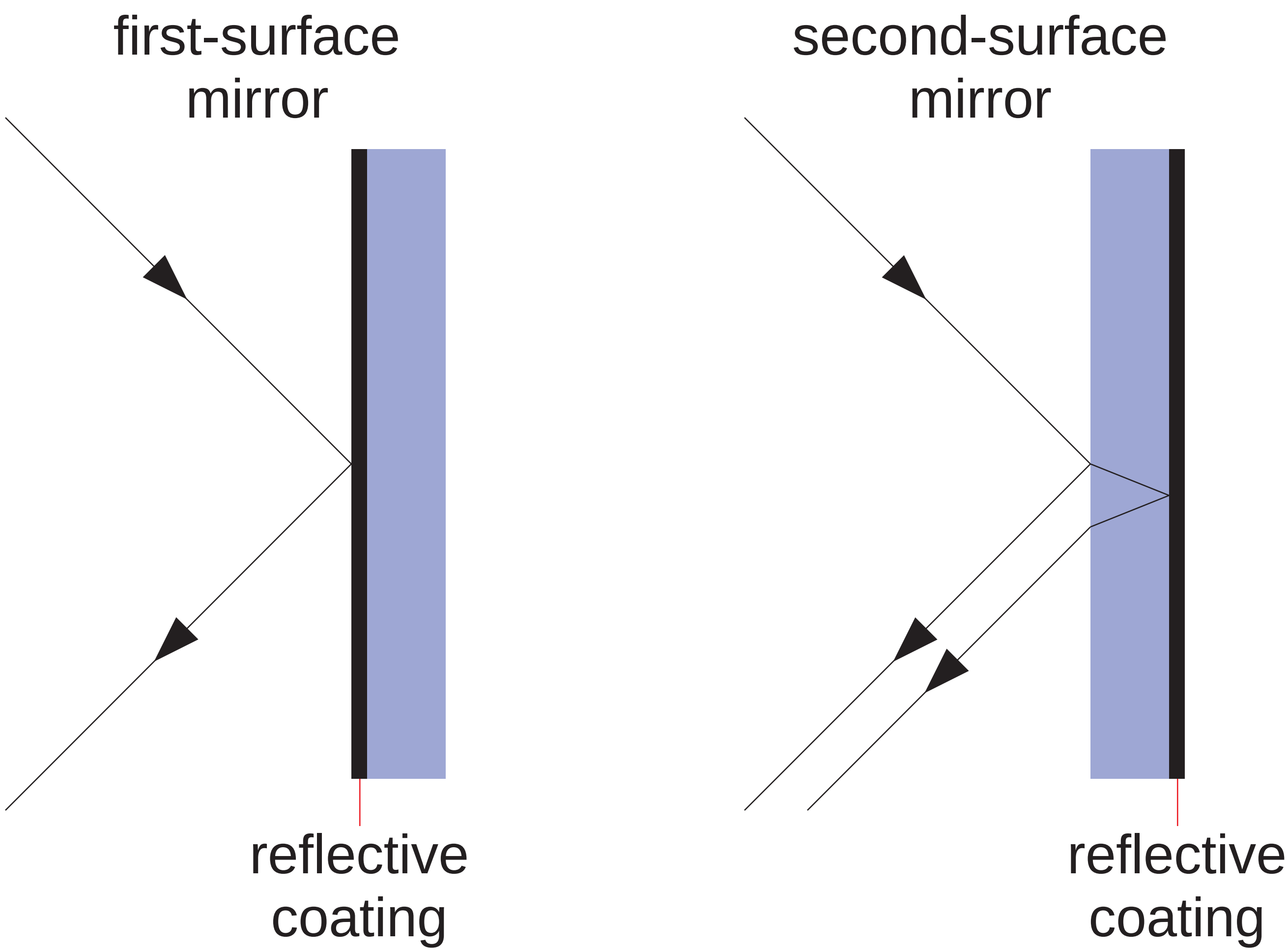
DISPERSION PROBLEM

Find the angular width $\Delta\delta$ of the dispersed beam exiting the triangular crown glass prism shown below. The incident ray represents white light.



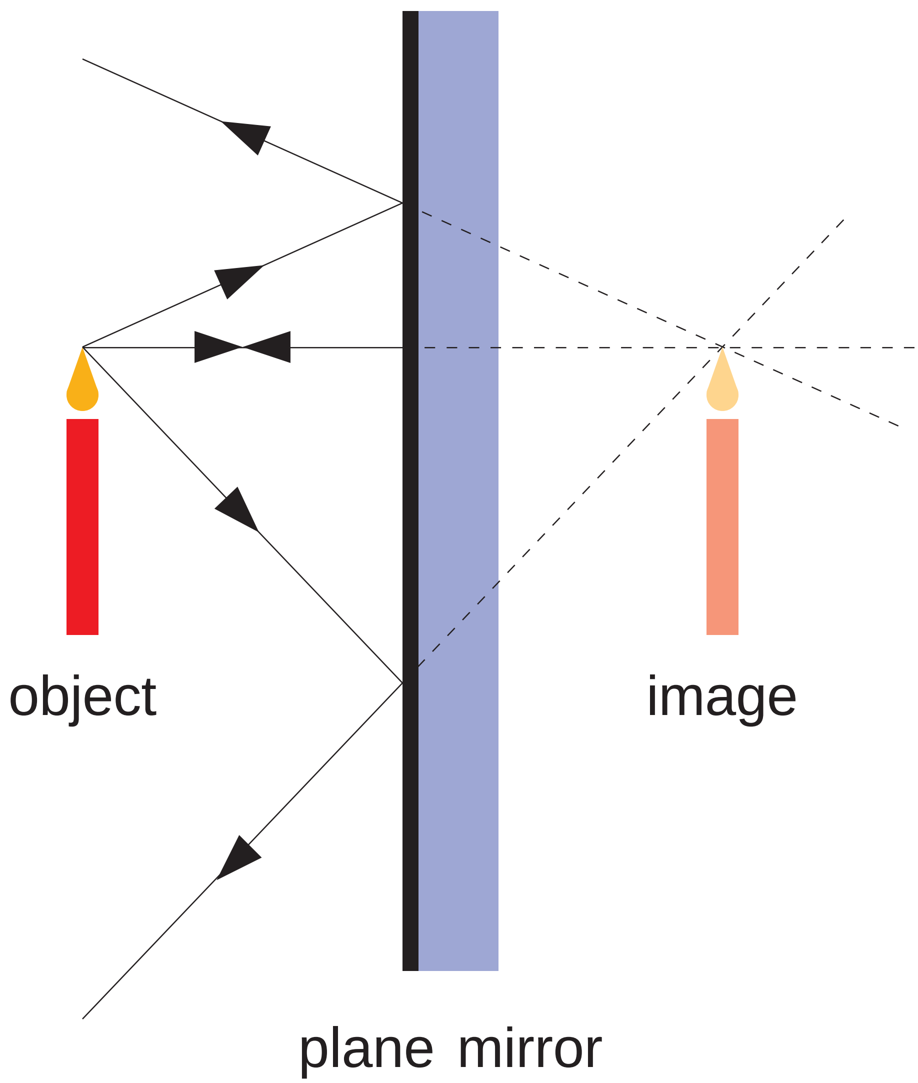
PLANE MIRRORS

A **plane mirror** is a portion of a planar surface with a reflective coating. Plane mirrors are usually constructed by placing a reflective coating of a material (such as aluminum or silver) on the front or rear surface of a planar slab of material (such as glass or plastic).



IMAGING BY PLANE MIRRORS

A **plane mirror** will always produce an image of an object that is: (1) as far behind the mirror as the object is in front of the mirror; (2) the same size as the object; (3) erect; and (4) virtual (located at a place where no rays pass).

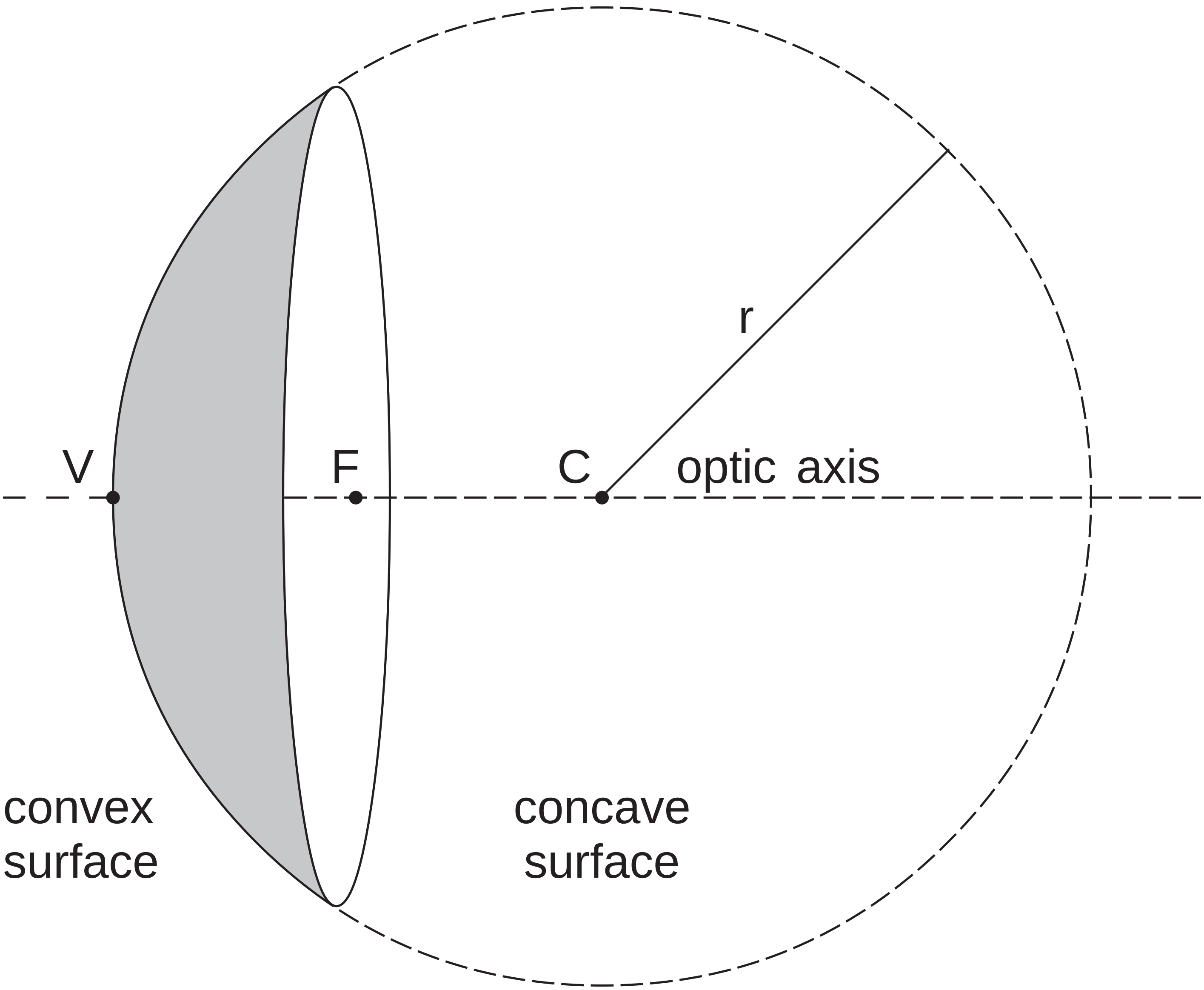


PLANE MIRROR PROBLEM

A girl walks into a dance studio. The east and west walls of the room are lined with plane mirrors. The girl is 15 feet from the east wall and 25 feet from the west wall. Describe the location of the images that she would see of herself (the distances that they appear to be from her) as she looked into both sets of mirrors.

SPHERICAL MIRRORS

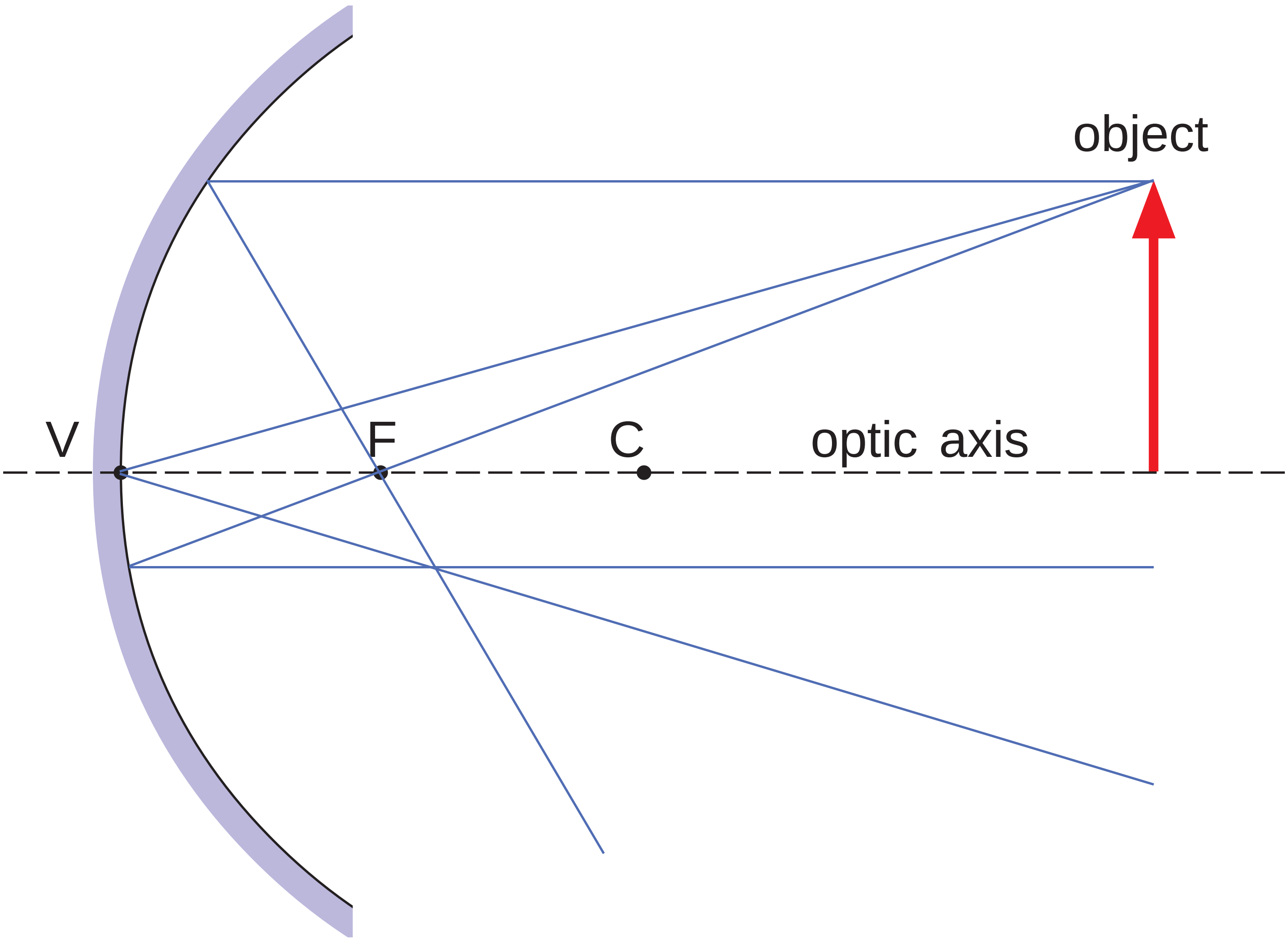
A **spherical mirror** is a portion of a spherical surface with a reflective coating. The spherical mirror is a concave (convex) spherical mirror if the reflective coating is applied to the concave (convex) surface.



V = vertex
F = focal point
C = center of curvature
r = radius of curvature

CONCAVE SPHERICAL MIRRORS

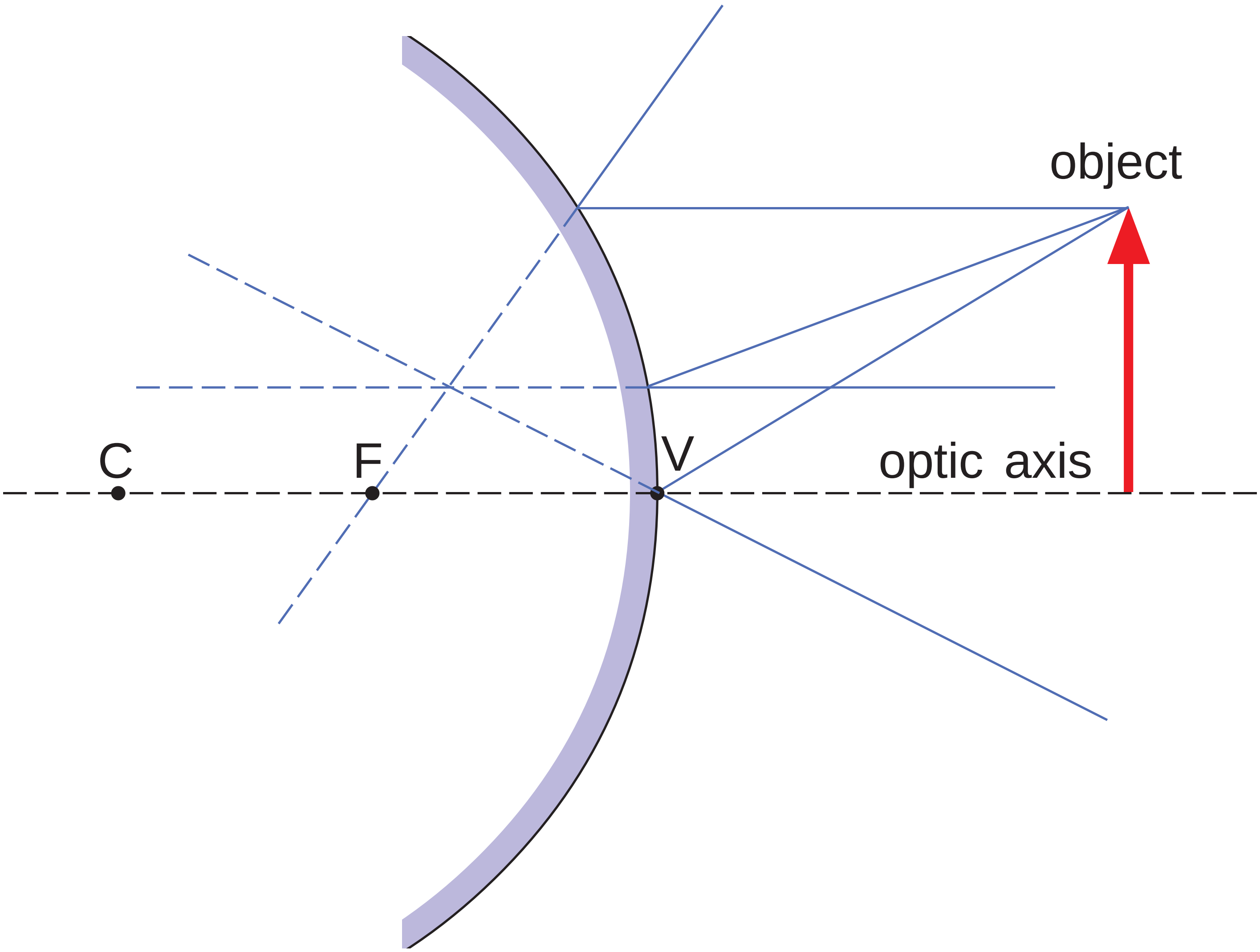
A **concave spherical mirror** reflects incident rays from an object such that:



- any incident ray passing through F is reflected as a paraxial ray
- any incident paraxial ray reflected is passed through F
- any incident ray striking V is reflected with mirror symmetry about the optic axis

CONVEX SPHERICAL MIRRORS

A **convex spherical mirror** reflects incident rays from an object such that:



- any incident ray directed toward F is reflected such that the reflected ray is a paraxial ray
- any incident paraxial ray is reflected such that its back-projected ray passes through F
- any incident ray striking V is reflected with mirror symmetry about the optic axis

IMAGE DESCRIPTION

The four pieces of information considered to completely define an image (for our purposes) are: (1) How far is the image from the mirror? (2) What side of the mirror is image on? (3) What is the size of the image relative to the object? (4) What is the orientation of the image? All four of these questions are answered by calculating two quantities: the image distance (q) and the lateral magnification (M).

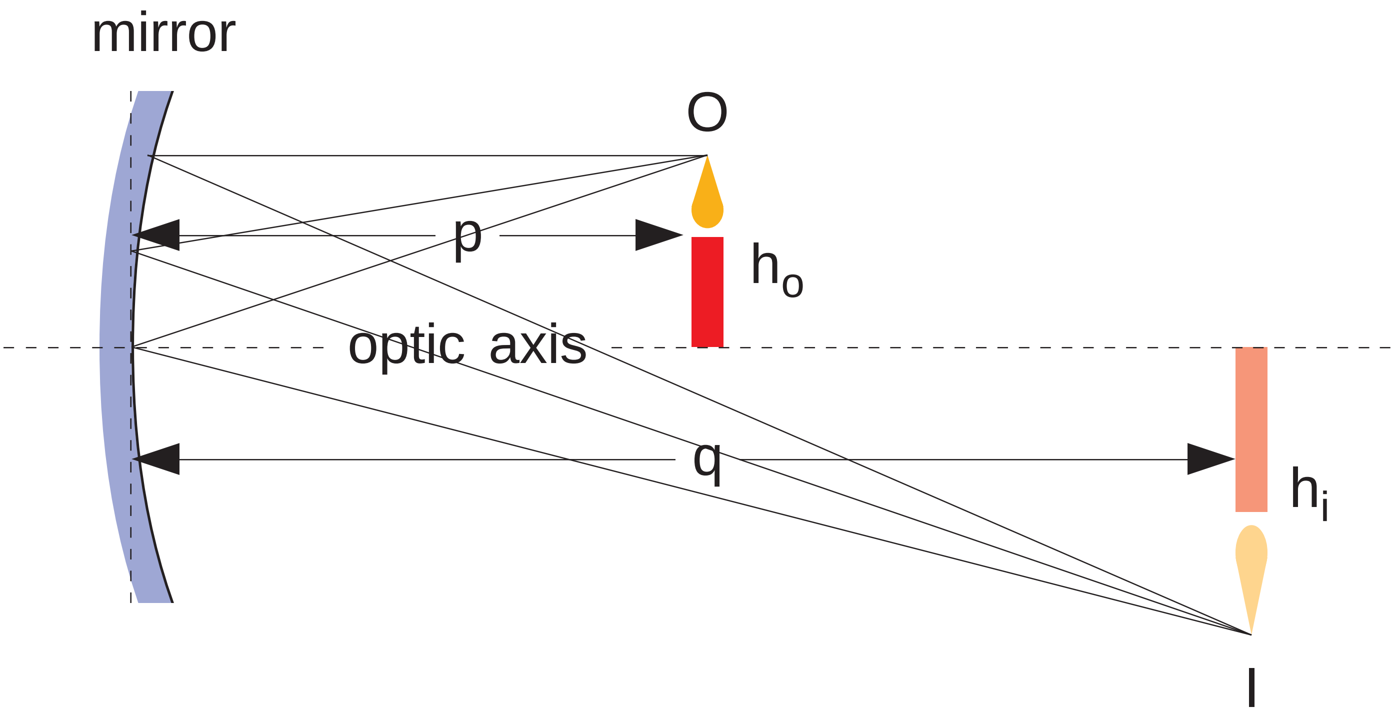


IMAGE PARAMETERS

p (object distance) = the distance the object is in front of the mirror (sign is positive)

q (image distance) = the signed distance the image is in from the mirror (if the image is in front [back] of the mirror, the sign of q is positive [negative]).

f (focal length of the mirror) = the distance between the vertex and the focal point of the mirror

h_o (object height) = lateral height of the object

h_i (image height) = lateral height of the image

THIN LENS EQUATION

The **thin lens equation** relates the distance that an object is from a mirror (or lens) and the focal length of the mirror (or lens) to the distance the image is located from the mirror (or lens).

$$1/f = 1/p + 1/q$$

other versions: $q = pf/(p-f)$

$$p = qf/(q-f)$$

$$f = pq/(p+q)$$

SPHERICAL MIRROR PROBLEM

A concave spherical mirror has a radius of curvature of 30.0 centimeters. If an object is placed 45.0 centimeters in front of the mirror, describe the image that is formed.

SPHERICAL MIRROR PROBLEM

A spherical mirror is used to produce an image of an object. When the object is placed 80.0 centimeters in front of the mirror, it is found that the mirror produces a real inverted image 100.0 centimeters in front of the mirror. What is the focal length of the mirror? Is the mirror a convex or concave mirror?

SPHERICAL MIRROR PROBLEM

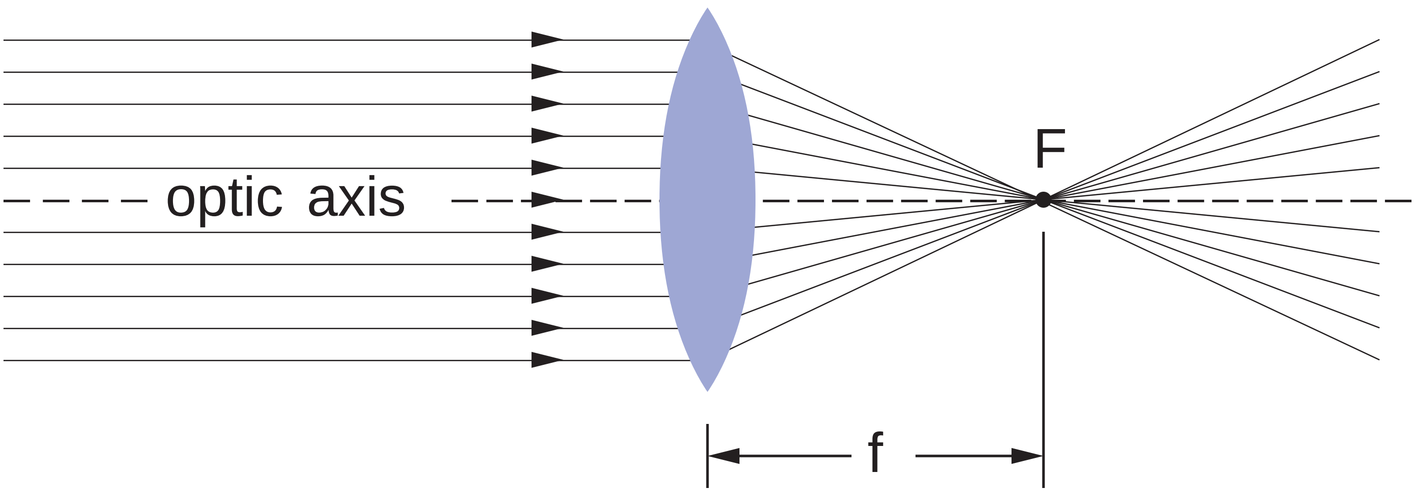
Suppose you were to look into a convex spherical mirror whose radius of curvature was 2.00 meters. How far away from you would your image appear to be if you were 1.50 meters in front of the mirror? Describe the image formed in this instance.

SPHERICAL MIRROR PROBLEM

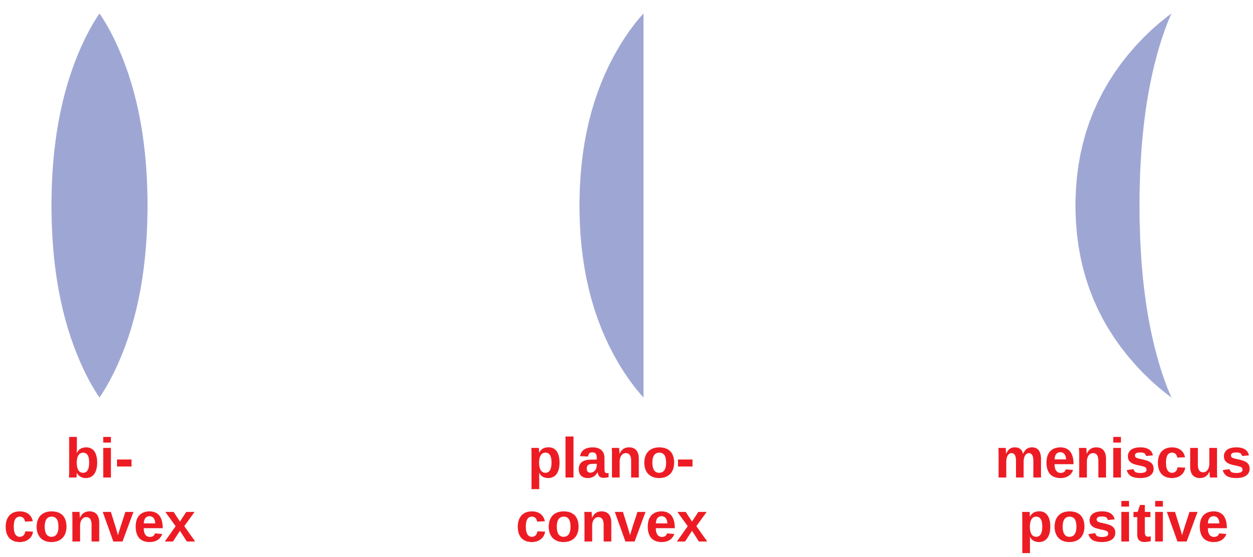
A 4-inch by 6-inch index card is placed in front of a concave spherical mirror whose focal length is 18.0 inches. The card is placed so that its long dimension lies along the optic axis. The 4-inch edge nearest the mirror is located a distance of 24.0 inches from the vertex of the mirror. Describe the size and shape of the image formed of the card.

CONVERGING LENSES

A **converging lens** is a lens that causes an incident beam of parallel rays to be converged through a point (called the **focal point** F). The **focal length** f of the lens is the positive distance between the midplane of the lens and the focal point.

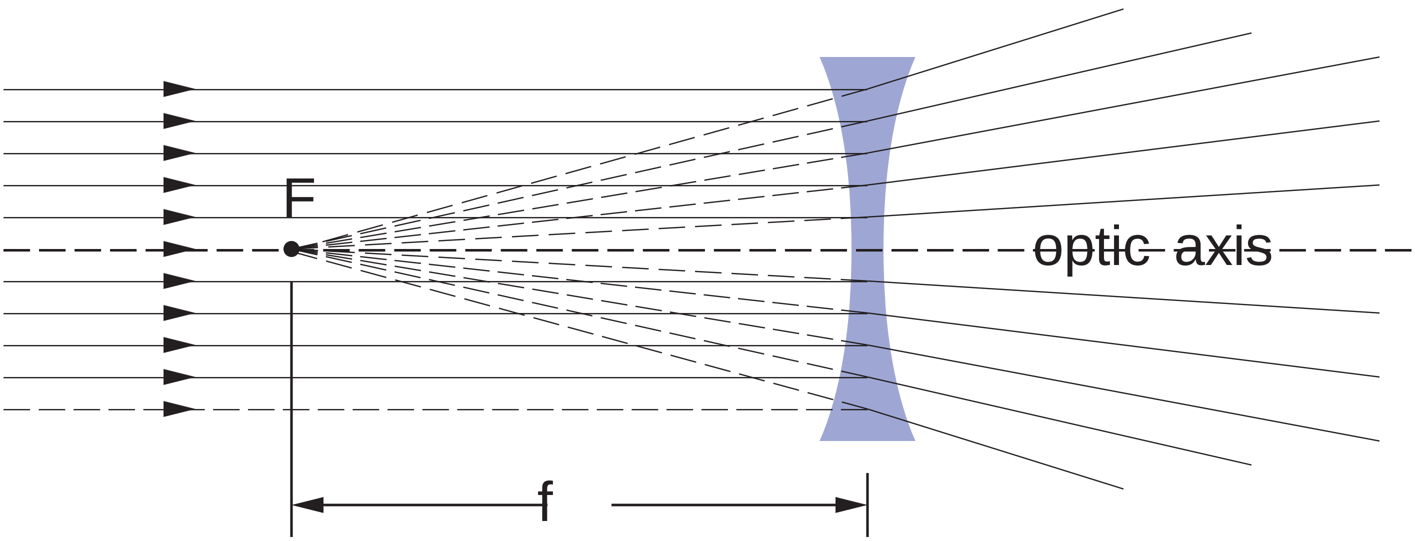


basic types of **converging** spherical lenses

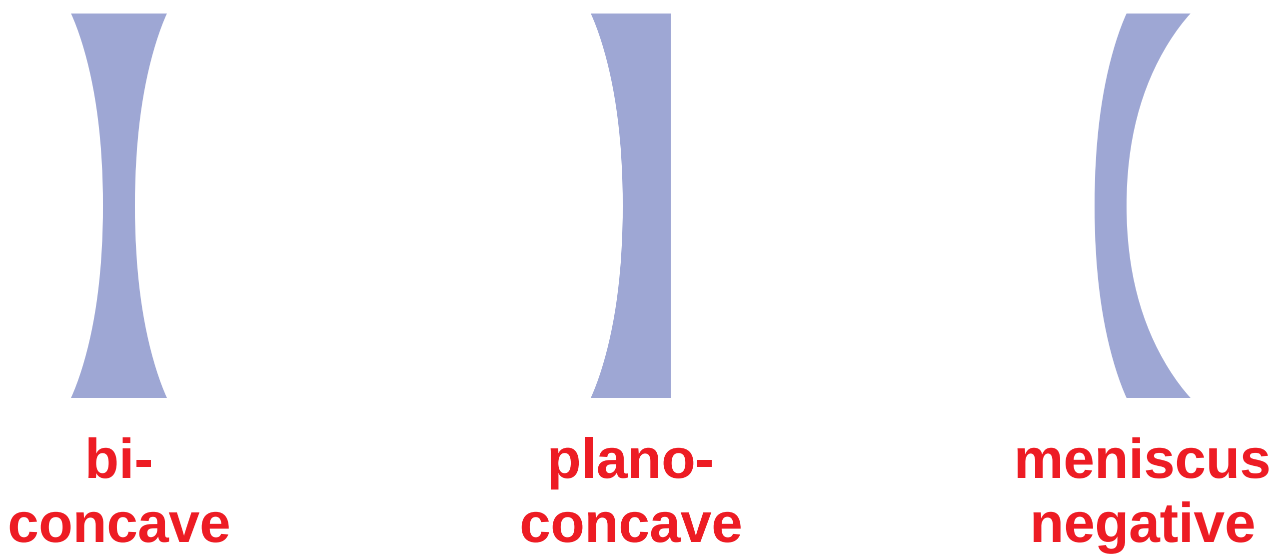


DIVERGING LENSES

A **diverging lens** is a lens that causes an incident beam of parallel rays to be diverged in such a way that the back-projected rays pass through a point (called the **focal point** F). The **focal length** f of the lens is the negative distance between the midplane of the lens and the focal point.

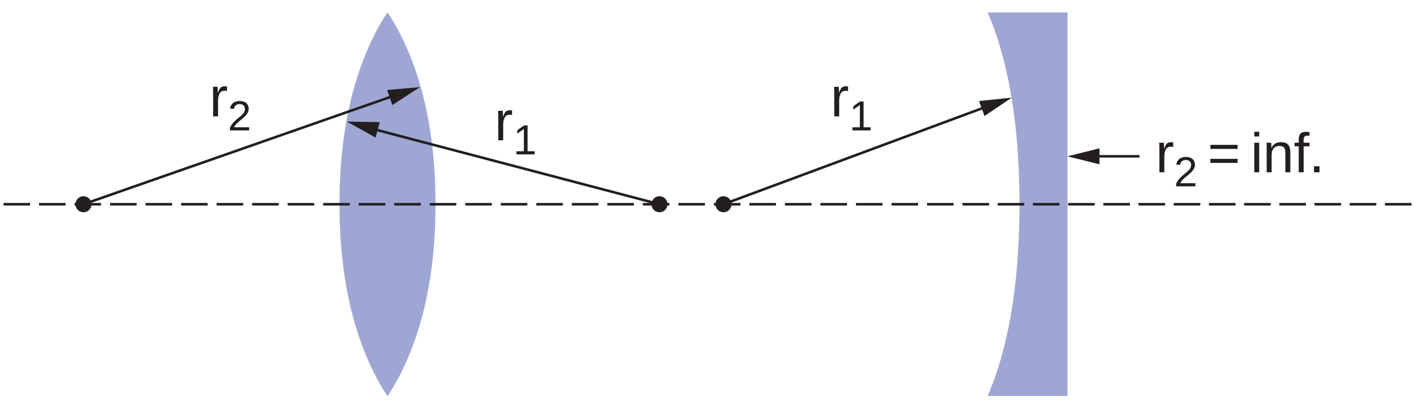


basic types of **diverging** spherical lenses



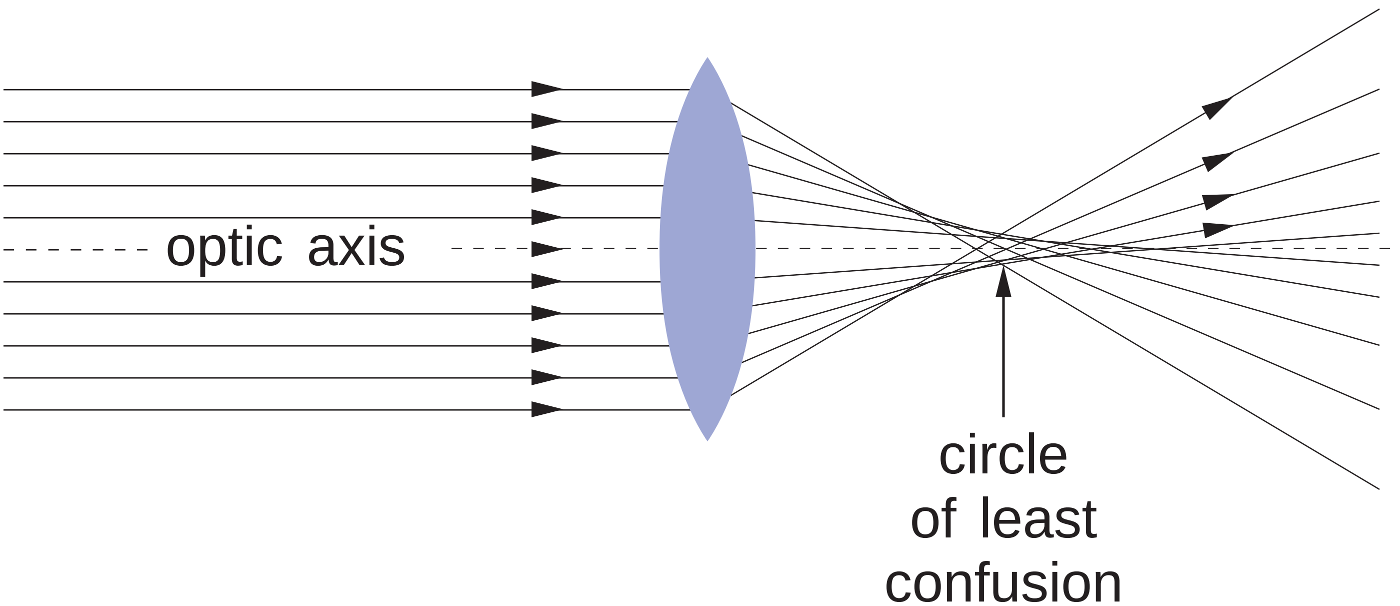
SPHERICAL LENSES

A **lens** is a transparent optical element that converges or diverges a beam of parallel rays. A **spherical lens** is a lens whose two surfaces are portions of spherical surfaces. The radii of curvature of the two surfaces may be the same, or different. A planar surface is considered to be a spherical surface whose radius of curvature is infinite.



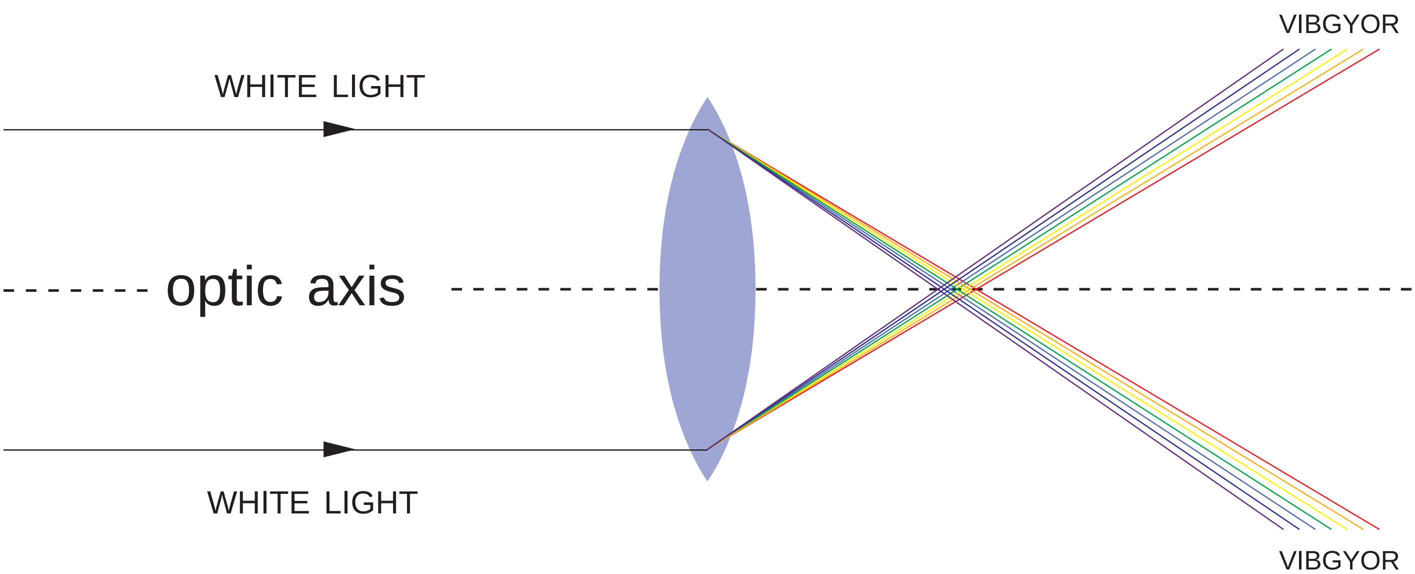
SPHERICAL ABERRATION

Spherical aberration is an aberration in which not all paraxial rays are brought through the same focal point. All spherical simple lenses suffer from this. The effect is less pronounced the "thinner" the lens is. Typically, innermore paraxial rays have longer focal lengths than outermore paraxial rays.



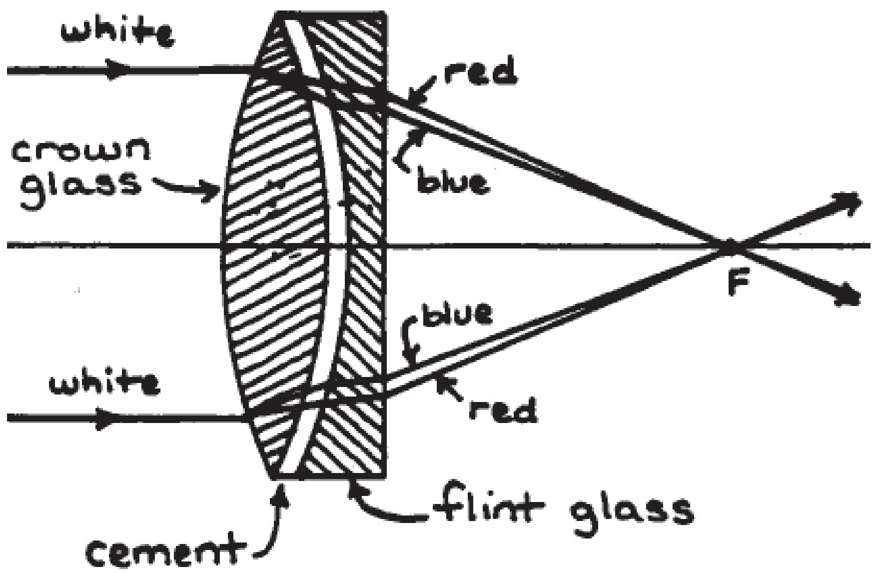
CHROMATIC ABERRATION

An aberration in which not all paraxial rays are brought through the same focal point. All simple lenses made of a single optical substance suffer from this. The effect is less pronounced the "thinner" the lens is, or the less dispersive the substance is. For many materials, violet light has a shorter focal length (higher n) than red light.



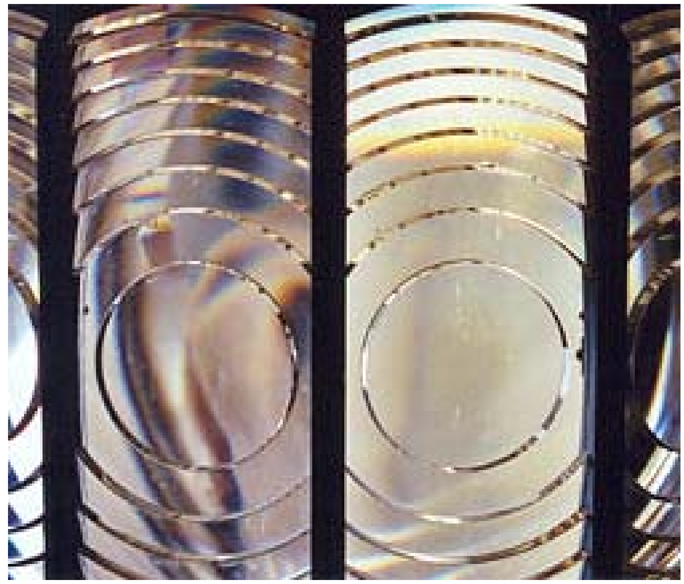
ACHROMATIC LENS

A compound lens used to correct chromatic aberration. It usually consists of a converging crown glass lens in contact (cemented to) a diverging flint glass lens. The chromatic aberration from each lens is used to "cancel out" the aberration from the combination. If two lenses are used, it is called a **doublet**. If three are used, it is called a **triplet**.

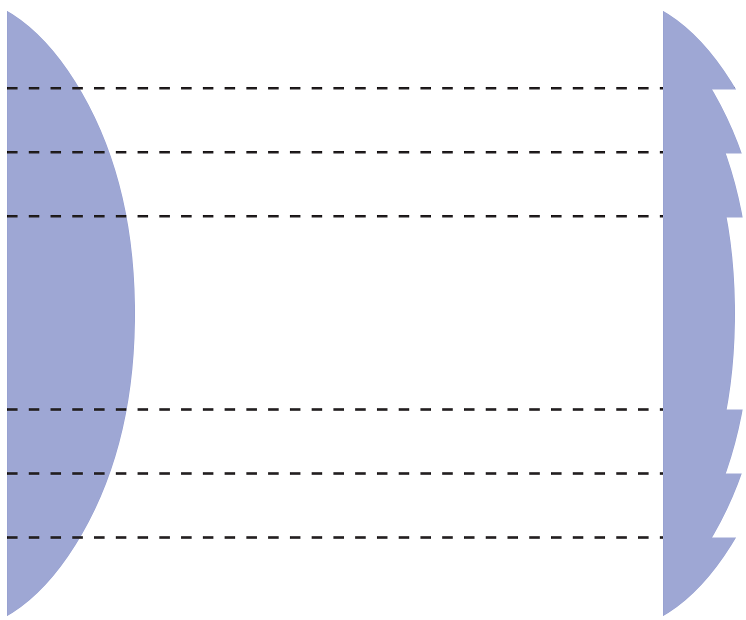


FRESNEL LENS

A lens that utilizes annular stepped zones to reduce bulk of optical medium (glass) in a lens, while maintaining its functionality. Greatly reduces the amount of material used in otherwise large, bulky lenses. Developed by A. Fresnel (French physicist).



lighthouse Fresnel lenses



conventional lens

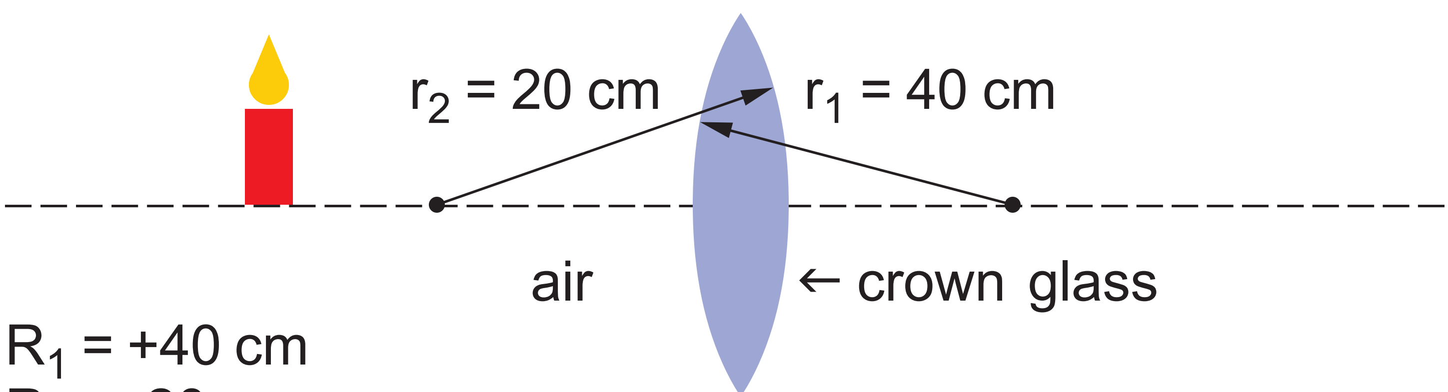
Fresnel lens

LENS MAKER'S FORMULA

The **lens maker's formula** is a formula that allows the computation of the focal length of a spherical lens given: (1) the signed radii of curvature of the spherical surfaces of the lens, (2) the index of refraction the lens, and (3) the index of refraction of the medium in which the lens is immersed. The formula specific to air being the medium in which the lens is immersed is:

$$f = \frac{R_1 R_2}{(n-1)(R_2 - R_1)}$$

example:



$R_1 = +40$ cm
$R_2 = -20$ cm

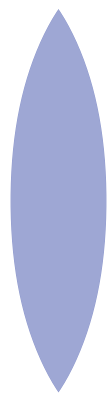
$$f = \frac{(40\text{ cm})(-20\text{ cm})}{(1.52 - 1)(-20\text{ cm} - 40\text{ cm})} = \boxed{+25.6\text{ cm}}$$

POWER OF A LENS

The **power P** of a lens is the reciprocal of its focal length, which is typically expressed in diopters (reciprocal meters). The diopter is abbreviated D ($1\ D = m^{-1}$).

example:

f = 80 cm ↘



$P = 1/(.8\ m)$

$= +1.25\ D$

note: In terms of power, the lens maker's formula can be expressed

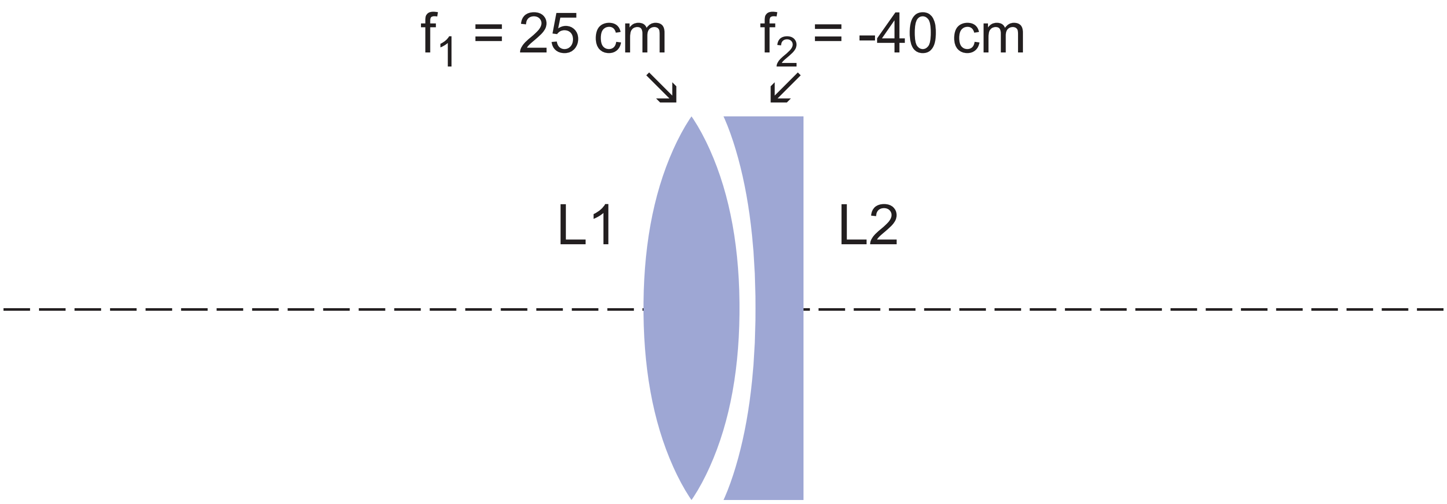
$$P = (n - 1)[1/R_1 - 1/R_2]$$

THIN LENSES IN CONTACT

When stacked as shown, thin lenses L1 and L2, with focal lengths f_1 and f_2 respectively, act as a single thin lens whose focal length f_{12} is given by:

$$f_{12} = \frac{f_1 f_2}{(f_1 + f_2)}$$

example:



$$f = \frac{(25\text{ cm})(-40\text{ cm})}{(25\text{ cm} + -40\text{ cm})} = \boxed{+66.7\text{ cm}}$$